## CHAPTER 11

# USE OF SEXTANT IN PILOTING 

## FUNDAMENTAL CONCEPTS

## 1100. Introduction

The marine sextant has long been an accurate means for fixing a vessel's position in coastal and confined water circumstances. However, with the advent of reliable gyrocompass technologies, followed by the introduction of precise electronic positioning systems like GPS, use of the marine sextant for terrestrial navigation has declined to such an extent that it is seldom employed during normal piloting conditions. This is unfortunate because the sextant can be used to great advantage in situations where other methods or tools, including the gyrocompass, are inadequate. The applications of the sextant during daylight in coastal waters, harbor approaches, and more confined waters may be summarized as follows:

1. fixing to make a safe transit of hazardous waters;
2. fixing to take a specific geographic position;
3. fixing to establish accurately the position of the anchor on anchoring;
4. fixing to determine whether or not the ship is dragging anchor;
5. using horizontal and vertical danger angles;
6. using vertical angles to determine distance off;
7. fixing to determine the positions of uncharted objects, or to verify the positions of charted features;
8. using the sextant to validate the accuracy of navigation by other means.

Because the use of the sextant has declined, many navigators, unfortunately, do not have the proficiency necessary to use it to advantage in those situations where other methods may be inadequate. Proficiency in the use of the sextant can be invaluable in situations where even a small error in either observing or plotting cross bearings could result in navigation blunder.

## 1101. Three-Point Problem

Normally, three charted objects are selected for measuring horizontal sextant angles to determine the observer's position, one of the objects being common to each angular measurement. With simultaneous or nearly simultaneous measurements of the horizontal angles between each pair of charted objects, the observer establishes two circles of position. For each pair of objects, there is only one circle which passes through the two objects and the observer's position.

Thus, there are two circles, intersecting at two points as shown in Figure 1101a, which pass through the observer's position at $T$.

Since the observer knows that $\mathrm{s} / \mathrm{he}$ is not at the intersection at $B, \mathrm{~s} /$ he must be at $T$.

The solution of what is known as the three-point problem is effected by placing the hairlines of the arms of a plastic three-arm protractor over the three observed objects on the chart as shown in Figure 1101b. With the arms so placed, the center of the protractor disk is over the observer's position on the chart at the time of the measurements.

## 1102. Solution Without Three-Arm Protractor

Although the conventional solution of the three-point problem is obtained by placing the arms of a three-arm protractor over the three observed objects on the chart, the use of the protractor is not necessary. The use of the protractor may not be practicable because of limited space and facilities for plotting, as in a small open boat. Where a common charted object cannot be used in the horizontal angle observations, a means other than the three-arm protractor must be employed to determine the position of the observer. Also, point fixes as obtained from the three-arm protractor can be misleading if the navigator has limited skill in evaluating the strengths of the three-point solutions.

In plotting the three-point fix without a three-arm protractor, the procedure is to find the center of each circle of position, sometimes called circle of equal angle (Figure 1102a), and then, about such center, to strike an arc of radius equal to the distance on the chart from the circle center to one of the two objects through which the circle passes. The same procedure is applied to the other pair of objects to establish the fix at the intersection of the two arcs.

Some of the methods for finding the center of a circle of equal angle are described in the following text.

The center of the circle of equal angle lies on the perpendicular bisector of the baseline of the pair of objects. With the bisector properly graduated (Figure 1102b), one need only to place one point of the compasses at the appropriate graduation, the other point at one of the observed objects, and then to strike the circle of equal angle or an arc of it in the vicinity of the DR.

The bisector can be graduated through calculation or by means of either the simple protractor or the three-arm protractor.


Figure 1101a. Solving the three-point problem.

As shown in Figure 1102a, when the observed angle is $90^{\circ}$, the center of the circle of equal angle lies at the center of the baseline or at the foot of the perpendicular bisector of the baseline. When the observed angle is less than $90^{\circ}$, for example $40^{\circ}$, the center of the circle lies on the perpendicular bisector on the same side of the baseline as the observer. When the observed angle is $26^{\circ} 34^{\prime}$, the center of the circle lies on the bisector at a distance from its foot equal to the distance between the two objects. When the observed angle is greater than $90^{\circ}$, the center of the circle lies on the perpendicular bisector on the side of the baseline opposite from the observer. The center for $100^{\circ}$ is the same distance from the baseline as the center for $80^{\circ}$; the center for $110^{\circ}$ is the same distance as the center for $70^{\circ}$, etc. These facts can be used to construct a nomogram for finding the distances of circles of equal angle from the foot of the perpendicular for various angles.

From geometry the central angle subtended by a chord is twice the angle with its vertex on the circle and subtended by the same chord. Therefore, when the observed horizontal angle is $30^{\circ}$, the central angle subtended by the baseline is $60^{\circ}$. Or, the angle at the center of the circle between the perpendicular bisector and the line in the direction of one of the observed objects is equal to the observed angle, or $30^{\circ}$ as shown in Figure 1102c. The angle
at the object between the baseline and the center of the circle on the bisector is $90^{\circ}$ minus observed angle, or $60^{\circ}$.

## 1103. Split Fix

Occasions when a common charted object cannot be used in horizontal angle observations are rare. On these occasions the mariner must obtain what is called a split fix through observation of two pairs of charted objects, with no object being common. As with the three-point fix, the mariner will obtain two circles of equal angle, intersecting at two points. As shown in Figure 1103, one of these two intersections will fix the observer's position.

## 1104. Conning Aid

Preconstructed circles of equal angle can be helpful in conning the vessel to a specific geographic position when fixing by horizontal angles. In one application, the vessel is conned to keep one angle constant, or nearly constant, in order to follow the circumference of the associated circle of equal angle to the desired position; the other angle is changing rapidly and is approaching the value for the second circle of equal angle passing through the desired position.


Figure 1101b. Use of the three-armed protractor.

## 1105. Strength of Three-Point Fix

Although an experienced navigator can readily estimate the strength of a three-point fix, and is able to select the objects providing the strongest fix available quickly, others often have difficulty in visualizing the problem and may select a weak fix when strong ones are available. The following generally useful (but not infallible) rules apply to selection of charted objects to be observed:

1. The strongest fix is obtained when the observer is inside the triangle formed by the three objects. And in such case the fix is strongest where the three objects form an equilateral triangle (Figure 1105, view A), the observer is at the center, and the objects are close to the observer.
2. The fix is strong when the sum of the two angles is equal to or greater than $180^{\circ}$ and neither angle is less than $30^{\circ}$. The nearer the angles are equal to each other, the stronger is the fix (view B).
3. The fix is strong when the three objects lie in a straight
line and the center object is nearest the observer (view C).
4. The fix is strong when the center object lies between the observer and a line joining the other two, and the center object is nearest the observer (view D).
5. The fix is strong when two objects a considerable distance apart are in range and the angle to the third object is greater than $45^{\circ}$ (view E).
6. Small angles should be avoided as they result in weak fixes in most cases and are difficult to plot. However, a strong fix is obtained when two objects are nearly in range and the nearest one is used as the common object. The small angle must be measured very accurately, and the position of the two objects in range must be very accurately plotted. Otherwise, large errors in position will result. Such fixes are strong only when the common object is nearest the observer. The fix will become very weak where the observer moves to a position where the distant object is the common object (view F).
7. A fix is strong when at least one of the angles changes rapidly as the vessel moves from one location to another.


Figure 1102a. Circles of equal angle.
8. The sum of the two angles should not be less than $50^{\circ}$; better results are obtained when neither angle is less than $30^{\circ}$.
9. Do not observe an angle between objects of considerably different elevation. Indefinite objects such as tangents, hill-tops, and other poorly defined or located points should not be used. Take care to select prominent objects such as major lights, church spires, towers or buildings which are charted and are readily distinguished from surrounding objects.

Beginners should demonstrate the validity of the above rules by plotting examples of each and their opposites. It should be noted that a fix is strong if, in plotting, a slight movement of the center of the protractor moves the arms away from one or more of the stations, and is weak if such movement does not appreciably change the relation of the arms to the three points. An appreciation of the accuracy required in measuring angles can be obtained by changing one angle about five minutes in arc in each example and noting the resulting shift in the plotted positions;

The error of the three-point fix will be due to:

1. error in measurement of the horizontal angles;
2. error resulting from observer and observed objects not lying in a horizontal plane;
3. instrument error; and
4. plotting error.

The magnitude of the error varies directly as the error in measurement, the distance of the common object from the observer, $(D)$ and inversely as the sine function of the angle of cut $(\theta)$. The magnitude of the error also depends upon the following ratios:

1. The distance to the object to the left of the observer divided by the distance from this object to the center object $\left(r_{1}\right)$.
2. The distance to the object to the right of the observer divided by the distance from this object to the center object ( $r_{2}$ ).

Assuming that each horizontal angle has the same error


Figure 1102b. Graduated perpendicular bisector.


Figure 1102c. Circle of equal angle (30 ${ }^{\circ}$.
$(\alpha)$, the magnitude of the error $(E)$ is expressed in the formula

$$
E=\frac{\alpha D}{\sin \theta} \sqrt{r_{1}^{2}+r_{2}^{2}+\left(2 r_{1}\right) r_{2} \cos \theta}
$$

where error in measurement $(\alpha)$ is expressed in radians.

The magnitude of the error $(E)$ is expressed in the formula

$$
E=\frac{0.00029 \alpha D}{\sin \theta} \sqrt{r_{1}^{2}+r_{2}^{2}+\left(2 r_{1}\right) r_{2} \cos \theta}
$$

where error in measurement $(\alpha)$ is expressed in minutes of
arc.
To avoid mistakes in the identification of charted objects observed, either a check bearing or a check angle should be used to insure that the objects used in observation and plotting are the same.

## 1106. Avoiding the Swinger

Avoid a selection of objects which will result in a "revolver" or "swinger"; that is, when the three objects observed on shore and the ship are all on, or near, the circumference of a circle (Figure 1106). In such a case the


Figure 1103. Split fix.
ship's position is indeterminate by three-point fix.
If bearings as plotted are affected by unknown and uncorrected compass error, the bearing lines may intersect at a point when the objects observed ashore and the ship are all on, or near, the circumference of a circle.

## 1107. Cutting in Uncharted Objects

To cut in or locate on the chart uncharted objects, such as newly discovered offshore wrecks or objects ashore which may be useful for future observations, proceed as follows:

1. Fix successive positions of the ship or ship's boat by three-point fixes, i.e., by horizontal sextant angles. At each fix, simultaneously measure the sextant angle between one of the objects used in the fix and the object to be charted (Figure 1107b). For more accurate results, the craft from which the observations are made should be either lying to or proceeding slowly.
2. For best results, the angles should be measured simultaneously. If verification is undertaken, the angles observed should be interchanged among observers.
3. The fix positions should be selected carefully to give strong fixes, and so that the cuts to the object will provide a good intersection at the next station taken for observations. A minimum of three cuts should be taken.

An alternative procedure is to select observing positions so that the object to be charted will be in range with one of the charted objects used to obtain the three-point fix
(Figure 1107a). The charted objects should be selected to provide the best possible intersections at the position of the uncharted object.

## 1108. Horizontal and Vertical Danger Angles

A vessel proceeding along a coast may be in safe water as long as it remains a minimum distance off the beach. This information may be provided by any means available. One method useful in avoiding particular dangers is the use of a danger angle. Refer to Figure 1108. A ship is proceeding along a coast on course line $A B$, and the captain wishes to remain outside a danger $D$. Prominent landmarks are located at $M$ and $N$. A circle is drawn through $M$ and $N$ and tangent to the outer edge of the danger. If $X$ is a point on this circle, angle $M X N$ is the same as at any other point on the circle (except that part between $M$ and $N$ ). Anywhere within the circle the angle is larger and anywhere outside the circle it is smaller. Therefore, any angle smaller than MXN indicates a safe position and any angle larger than $M X N$ indicates possible danger. Angle $M X N$ is therefore a maximum horizontal danger angle. A minimum horizontal danger angle is used when a vessel is to pass inside an offlying danger, as at $D^{\prime}$ in Figure 1108. In this case the circle is drawn through $M$ and $N$ and tangent to the inner edge of the danger area. The angle is kept larger than $M Y N$. If a vessel is to pass between two danger areas, as in Figure 1108, the horizontal angle should be kept smaller than MXN but larger than $M Y N$. The minimum danger angle is effective


View A


View C


View E


View B


View F

Figure 1105. Strengths of three-point fixes.
only while the vessel is inside the larger circle through $M$ and $N$. Bearings on either landmark might be used to indicate the entering and leaving of the larger circle. A margin of safety can be provided by drawing the circles through points a short distance off the dangers. Any method of measuring the angles, or difference of bearing of $M$ and $N$, can be used. Perhaps the most accurate is by horizontal sextant angle. If a single landmark of known height is available, similar procedure can be used with a vertical danger angle between top and bottom of the object. In this case the charted position of the object is used as the center of the circles.

## 1109. Distance by Vertical Angle

Table 16 (Distance by Vertical Angle) provides means for determining the distance of an object of known height above sea level. The vertical angle between the top of the object and the visible (sea) horizon (the sextant altitude) is measured and corrected for index error and dip only. If the visible horizon is not available as a reference, the angle should be measured to the bottom of the object, and dip short of the horizon (Table 14) used in place of the usual dip correction. This may require several approximations of distance


Figure 1106. Revolver or swinger.


Figure 1107a. On range method.
by alternate entries of tables 16 and 15 until the same value is obtained twice. The table is entered with the difference in the height of the object and the height of eye of the observer, in feet, and the corrected vertical angle; and the distance in nautical miles is taken directly from the table. An error may be introduced if refraction differs from the standard value used in the computation of the table. See the Explanation of Tables section in Volume II for more details.

## 1110. Evaluation

As time and conditions permit, it behooves the navigator to use the sextant to evaluate the accuracy of navigation by other means in pilot waters. Such accuracy comparisons tend to provide navigators with better appreciation of the limitations of fixing by various methods in a given piloting situation.


SHIP STATIONS

Figure 1107b. Cutting in uncharted objects.


Figure 1108. Horizontal danger angles.

