

CHAPTER 2

INTERPOLATION

FINDING THE VALUE BETWEEN TABULATED ENTRIES

200. Introduction

When one quantity varies with changing values of a second quantity, and the mathematical relationship of the two is known, a curve can be drawn to represent the values of one corresponding to various values of the other. To find the value of either quantity corresponding to a given value of the other, one finds that point, on the curve defined by the given value, and reads the answer on the scale relating to the other quantity. This assumes, of course, that for each value of one quantity, there is only one value of the other quantity.

Information of this kind can also be tabulated. Each entry represents one point on the curve. The finding of value *between* tabulated entries is called **interpolation**. The extending of tabulated values to find values *beyond* the limits of the table is called **extrapolation**.

Thus, the *Nautical Almanac* tabulates values of declination of the sun for each hour of Coordinated Universal Time (UTC) or Universal Time (UT). The finding of declination for a time between two whole hours requires interpolation. Since there is only one entering **argument** (in this case UT), **single interpolation** is involved.

Table 11 gives the distance traveled in various times at certain speeds. In this table there are two entering arguments. If both given values are between tabulated values, **double interpolation** is needed.

In *Pub. No. 229*, azimuth angle varies with a change in any of the three variables: latitude, declination, and local hour angle. With intermediate values of all three, **triple interpolation** is needed.

Interpolation can sometimes be avoided. A table having a single entering argument can be arranged as a **critical table**. An example is the dip (height of eye) correction on the inside front cover of the *Nautical Almanac*. Interpolation is avoided through dividing the argument into intervals so chosen that successive intervals correspond to successive values of the required quantity, the respondent. For any

value of the argument within these intervals, the respondent can be extracted from the table without interpolation. The lower and upper limits (critical values) of the argument correspond to half-way values of the respondent and, by convention, are chosen so that when the argument is equal to one of the critical values, the respondent corresponding to the preceding (upper) interval is to be used. Another way of avoiding interpolation would be to include every possible entering argument. If this were done for *Pub. No. 229*, interpolation being eliminated for declination only, and assuming declination values to 0'.1, the number of volumes would be increased from six to more than 3,600. If interpolation for meridian angle and latitude, to 0'.1, were also to be avoided, a total of more than 1,296,000,000 volumes would be needed. A more practical method is to select an assumed position to avoid the need for interpolation for two of the variables.

201. Single Interpolation

The accurate determination of intermediate values requires knowledge of the nature of the change between tabulated values. The simplest relationship is **linear**, the change in the tabulated value being directly proportional to the change in the entering argument. Thus, if a vessel is proceeding at 15 knots, the distance traveled is directly proportional to the time, as shown in Figure 201a. The same information might be given in tabular form, as shown in table 201b. Mathematically, this relationship for 15 knots is written $D = \frac{15t}{60} = \frac{t}{4}$, where D is distance in nautical miles, and t is time in minutes.

In such a table, interpolation can be accomplished by simple proportion. Suppose, for example, that the distance is desired for a time of 15 minutes. It will be some value between 3.0 and 4.0 miles, because these are the distances for 12 and 16 minutes, respectively, the tabulated times on each

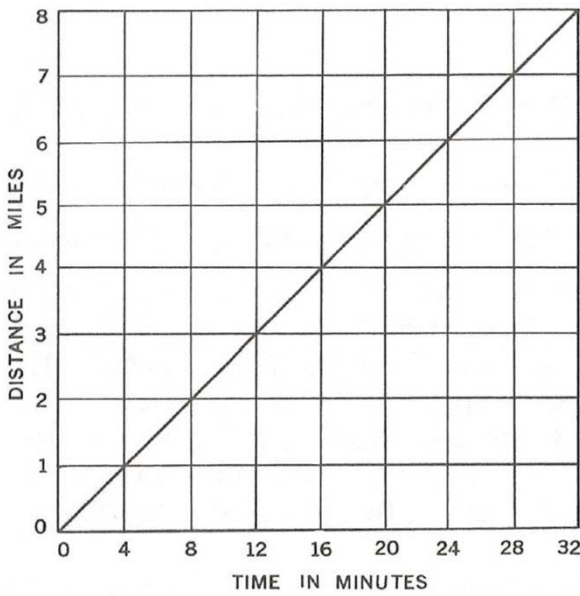


Figure 201a. Plot of $D = t / 4$.

side of the desired time.

Minutes	Miles
0	0.0
4	1.0
8	2.0
12	3.0
16	4.0
20	5.0
24	6.0
28	7.0
32	8.0

Table 201b. Table of $D = t / 4$

The proportion might be formed as follows:

$$\begin{array}{l} 3 \left[\begin{array}{l} 12 \\ 15 \\ 16 \end{array} \right] 4 \\ x \left[\begin{array}{l} 3.0 \\ y \\ 4.0 \end{array} \right] 1.0 \end{array}$$

$$\frac{3}{4} = \frac{x}{1.0}$$

$$x = \frac{3 \cdot 1.0}{4} = 0.75 \text{ (0.8 to the nearest 0.1 mi.)}$$

$$y = 3.0 + x = 3.0 + 0.8 = 3.8 \text{ mi.}$$

A simple interpolation such as this should be performed mentally. During the four-minute interval between 12 and 16 minutes, the distance *increases* 1.0 mile from 3.0 to 4.0 miles. At 15 minutes, $\frac{3}{4}$ of the interval has elapsed, and so the distance *increases* $\frac{3}{4}$ of 1.0 mile, or 0.75 mile, and is therefore $3.0 + 0.8 = 3.8$, to the nearest 0.1 mile.

This might also have been performed by starting with 16 minutes, as follows:

$$1 \left[\begin{array}{l} 12 \\ 15 \\ 16 \end{array} \right] 4$$

$$(-)x \left[\begin{array}{l} 3.0 \\ y \\ 4.0 \end{array} \right] 1.0$$

$$\frac{1}{4} = \frac{(-)x}{1.0}$$

$$x = (-)0.25 \text{ (-0.2 to the nearest 0.1 mi.)}$$

$$y = 4.0 - 0.2 = 3.8$$

Mentally, 15 is one quarter of the way from 16 to 12, and therefore the distance is $\frac{1}{4}$ the way between 4.0 and 3.0, or 3.8.

This interpolation might have been performed by noting that if distance changes 1.0 mile in four minutes, it must change $\frac{1.0}{10} = 0.1$ mile in $\frac{4}{10} = 0.4$ minute, or 24 seconds.

This relationship can be used for mental interpolation in situations which might seem to require pencil and paper. Thus, if distance to the nearest 0.1 mile is desired for 13m 15s, the answer is 3.3 miles, determined as follows: The time 13^m15^s is 1^m15^s (1.2^m approx.) more than 12^m. If 1.2 is divided by 0.4, the quotient is 3, to the nearest whole number. Therefore, $3 \cdot 0.1 = 0.3$ is added to 3, the tabulated value for 12 minutes. Alternatively, 13^m15^s is 2^m45^s (2.8m approx.) less than 16^m, and $2.8 \cdot 0.4 = 1.12$, and therefore the interpolated value is $7 - 0.1 = 6.9$ less than 4, the tabulated value for 16^m. In either case, the interpolated value is 3.3 miles.

A common mistake in single interpolation is to apply the correction (x) with the wrong sign, particularly when it should be negative (-). This mistake can be avoided by always checking to be certain that the interpolated value lies between the two values used in the interpolation.

When the curve representing the values of a table is a straight line, as in a, the process of finding intermediate values in the manner described above is called **linear interpolation**. If tabulated values of such a line are exact (not approximations), as in Table 201b, the interpolation can be carried to any degree of precision without sacrificing accuracy. Thus, in 21.5 minutes the distance is

$5.0 + \frac{1.5}{4} \cdot 1.0 = 5.375$ miles. Similarly, for 29.9364 minutes the distance is $7.0 + \frac{1.9364}{4} \cdot 1.0 = 7.4841$ miles, a value which has little or no significance in practical navigation. If one had occasion to find such a value, it could most easily be done by dividing the time, in minutes, by 4, since the distance increases at the rate of one mile each four minutes. This would be a case of avoiding interpolation by solving the equation connecting the two quantities. For a simple relationship such as that involved here, such a solution might be easier than interpolation.

Many of the tables of navigation are not linear. Consider Figure 201b. From Table 24 (Altitude Factors) it is found that for latitude 25° and declination 8° , same name, the variation of altitude in one minute of time from meridian transit (the altitude factor) is $6.0''$ ($0.1'$). For limited angular distance on each side of the celestial meridian, the change in altitude is approximately equal to at^2 , where a is the altitude factor (from Table 24) and t is the time in minutes from meridian transit. Figure 201c is the plot of change in altitude against time. The same information is shown in tabular form in Table 201d.

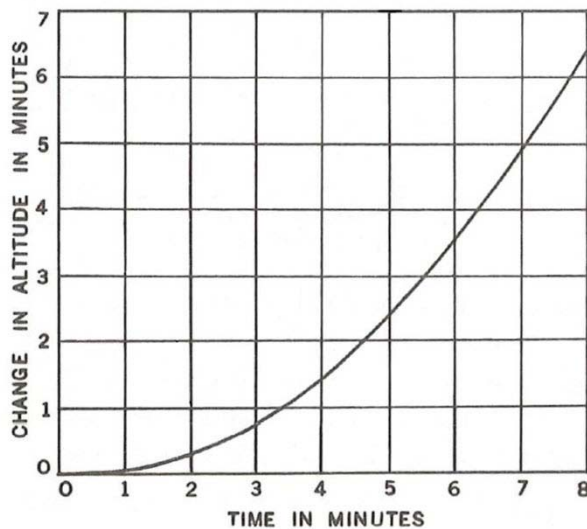


Figure 201c. Plot of altitude change = at^2 .

Minutes	Miles
0	0.0
1	0.1
2	0.4

Table 201d. Table of altitude change = at^2 , where $a=0.1'$.

3	0.9
4	1.6
5	2.5
6	3.6
7	4.9
8	6.4

Table 201d. Table of altitude change = at^2 , where $a=0.1'$.

To be strictly accurate in interpolating in such a table, one should consider the curvature of the line. However, in most navigational tables the points on the curve selected for tabulation are sufficiently close that the portion of the curve between entries can be considered a straight line without introducing a significant error. This is similar to considering the line of position from a celestial observation as a part of the circle of equal altitude. Thus, to the nearest $0.1'$, the change of altitude for 3.4 minutes is $0.9' + (0.4 \cdot 0.7') = 0.9' + 0.3' = 1.2'$. The correct value by solution of the formula is $1.156'$. The value for 6.8 minutes is $4.6'$ by interpolation and $4.624'$ by computation.

Section 204 (Nonlinear Interpolation) addresses the nonlinear interpolation used when the curve representing tabular values under consideration is not a close approximation to a straight line. However, such instances are infrequent in navigation, and generally occur at a part of the navigation table that is not commonly used, or for which special provisions are made. For example, in *Pub. No. 229* nonlinear interpolation *may* be required only when the altitude is above 60° . Even when the altitude is above 60° , the need for nonlinear interpolation is infrequent. When it is needed, such fact is indicated by the altitude difference being printed in italic type followed by a small dot.

202. Double Interpolation

In a double-entry table it may be necessary to interpolate for each entering argument. Table 202a is an extract from Table 22 (amplitudes). If one entering argument is an exact tabulated value, the amplitude can be found by single interpolation. For instance, if latitude is 45° and declination is 21.8° , amplitude is $31.2^\circ + \left(\frac{3}{5} \cdot 0.8^\circ\right) = 31.2^\circ + 0.5^\circ = 31.7^\circ$. However, if neither entering argument is a tabulated value, double interpolation is needed. This may be accomplished in any of several ways:

Lat.	Declination	
	21.5°	22.0°

Table 202a. Excerpts from amplitude table.

°	°	°
45	31.2	32.0
46	31.8	32.6

Table 202a. Excerpts from amplitude table.

“Horizontal” method. Use single interpolation for declination for each tabulated value of latitude, followed by single interpolation for latitude. Suppose latitude is 45.7° and declination is 21.8°. First, find the amplitude for latitude 45°, declination 21.8°, as above, 31.7°. Next, repeat the process for latitude 46°: $31.8^\circ + \left(\frac{3}{5} 0.8^\circ\right) = 32.3^\circ$. Finally, interpolate between 31.7° and 32.3° for latitude 45.7°: $31.7^\circ + (0.7 0.6^\circ) = 32.1^\circ$. This is the equivalent of first inserting a new column for declination 21.8°, followed by single interpolation in this column, as shown in Table 202b.

Lat.	Declination		
	21.5°	21.8°	22.0°
°	°	°	°
45	31.2	31.7	32.0
45.7		32.1	
46	31.8	32.3	32.6

Table 202b. “Horizontal” method of double interpolation.

“Vertical” method. Use single interpolation for latitude for each tabulated value of declination, followed by single interpolation for declination. Consider the same example as above. First, find the amplitude for declination 21.5°, latitude 45.7°: $31.2^\circ + (0.7^\circ 0.6^\circ) = 31.6^\circ$. Next, repeat the process for declination 22.0°: $32.0^\circ + (0.7^\circ 0.6^\circ) = 32.4^\circ$. Finally, interpolate between 31.6° and 32.4° for declination 21.8°: $31.6^\circ + \left(\frac{3}{5} 0.8^\circ\right) = 32.1^\circ$. This is the equivalent of first inserting a new line for latitude 45.7°, followed by single interpolation in this line, as shown in Table 202c.

Lat.	Declination		
	21.5°	21.8°	22.0°
°	°	°	°

Table 202c. “Vertical” method of double interpolation.

45	31.2		32.0
45.7	31.6	32.1	32.4
46	31.8		32.6

Table 202c. “Vertical” method of double interpolation.

“Combined” method. Select a tabulated “base” value, preferably that nearest the given tabulated entering arguments. Next, find the correction to be applied, with its sign, for single interpolation of this base value both horizontally and vertically. Finally, add these two corrections algebraically and apply the result, in accordance with its sign, to the base value. In the example given above, the base value is 32.6°, for declination 22.0° (21.8° is nearer 22.0° than 21.5°) and latitude 46° (45.7° is nearer 46° than 45°). The correction for declination is $\frac{2}{5} (-)0.8^\circ = (-)0.3^\circ$. The correction for latitude is $0.3^\circ (-)0.6^\circ = (-)0.2^\circ$. The algebraic sum is $(-)0.3^\circ + (-)0.2^\circ = (-)0.5^\circ$. The interpolated value is then $32.6^\circ - 0.5^\circ = 32.1^\circ$. This is the method customarily used by navigators, however, it is also less precise. If more accuracy is required more tedium must be exercised using the horizontal or vertical methods.

203. Triple Interpolation

With three entering arguments, the process is similar to that for double interpolation. It would be possible to perform double interpolation for the tabulated value on each side of the given value of one argument, and then interpolate for that argument, but the method would be tedious. The only method commonly used by navigators is that of selecting base value and applying corrections.

204. Nonlinear Interpolation

When the curve representing the values of a table is nearly a straight line, or the portion of the curve under consideration is nearly a straight line, linear interpolation suffices. However, when the successive tabular values are so nonlinear that a portion of the curve under consideration is not a close approximation to a straight line, it is necessary to include the effects of second differences, and possibly higher differences, as well as first differences in the interpolation.

The plot of Table 204b data in Figure 204a indicates that the altitude does not change linearly between declination values of 51° and 52°. If the first difference only were used in the interpolation, the interpolated value of altitude would lie on the straight line between points on the curve for declination values of 51° and 52°.

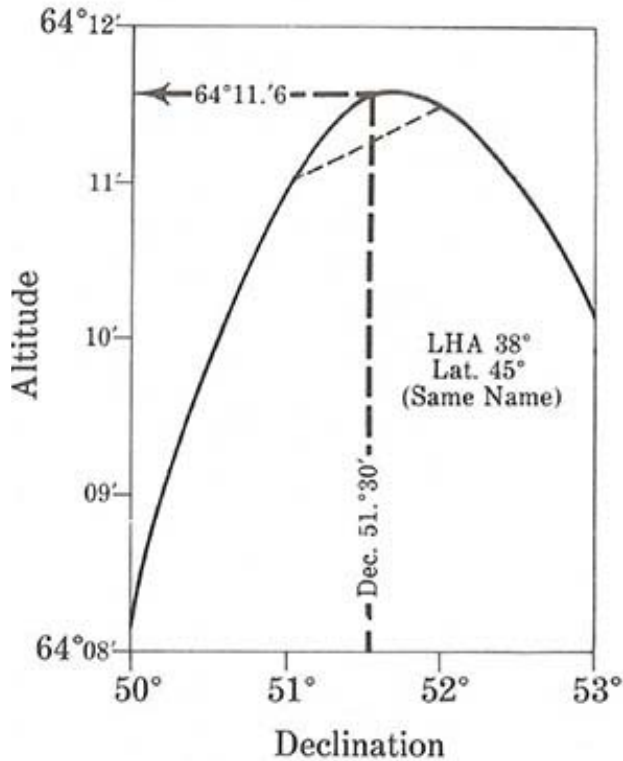


Figure 204a. Altitude curve.

LHA 38°, Lat. 45° (Same as Dec.)			
Dec.	ht (Tab. Hc)	First Difference	Second Difference
50°	64°08.2'		
		+2.8'	
51°	64°11.0'		-2.3'
		+0.5'	
52°	64°11.5'		-2.1'
		-1.6'	
53°	64°09.9'		

Table 204b. Data from Pub. No. 229.

If the altitude for declination 51°30' is obtained using only the **first difference**, i.e., the difference between successive tabular altitudes in this case, $Hc = 64°11.0' + \frac{30'}{60'} \cdot 0.5' = 64°11.3'$. However, inspection of Figure 204a reveals that this interpolated altitude is

0.3' low. If the tabular data were such that the differences between successive first differences, the **second differences**, were nearly zero, interpolation using the first difference only would provide the correct altitude. In this case, however, second differences are significant and must be included in the interpolation.

Function	First Difference	Second Difference
f_{-2}		δ_{-2}^2
	$\delta_{-3/2}$	
f_{-1}		δ_{-1}^2
	$\delta_{-1/2}$	
f_0		δ_0^2
	$\delta_{1/2}$	
f_{+1}		δ_1^2
	$\delta_{3/2}$	
f_{+2}		δ_2^3

Table 204c. Notation used with Bessel's Formula.

Table 204c shows the format and notation used to distinguish the various tabular quantities and differences when using Bessel's formula for the nonlinear interpolation. The quantities $f_{-2}, f_{-1}, f_0, f_{+1}, f_{+2}, f_{+3}$ represent successive tabular values.

Allowing for first and second differences only, Bessel's formula is stated as:

$$f_p = f_0 + p\delta_{1/2} + B_2(\delta_0^2 + \delta_1^2)$$

In this case, f_p is the computed altitude; f_0 is the tabular altitude; p is the fraction of the interval between tabular values of declination. The quantity B_2 is a function of p and is always negative. This coefficient is tabulated in Table 204d. The quantity $(\delta_0^2 + \delta_1^2)$ is the **double second difference (DSD)**, which is the sum of successive second differences.

Applying Bessel's formula to the data of table 205a to obtain 'the altitude for a declination of 51°30',

$$f_p = f_0 + p\delta_{1/2} + B_2(\delta_0^2 + \delta_1^2)$$

$$Hc = 64^\circ 11.0' + \left(\frac{30'}{60'}\right)(0.5') + (-0.062)[-2.3' + (-2.1)']$$

$$Hc = 64^\circ 11.0' + 0.3' + 0.3' = 64^\circ 11.6'$$

<i>p</i>	<i>B</i> ₂	<i>p</i>	<i>B</i> ₂	<i>p</i>	<i>B</i> ₂	<i>p</i>	<i>B</i> ₂	<i>p</i>	<i>B</i> ₂
0.0000	.000	0.1101	.025	0.2719	.050	0.7280	.049	0.8898	.024
.0020	.001	.1152	.026	.2809	.051	.7366	.048	.8949	.023
.0060	.002	.1205	.027	.2902	.052	.7449	.047	.9000	.022
.0101	.003	.1258	.028	.3000	.053	.7529	.046	.9049	.021
.0142	.004	.1312	.029	.3102	.054	.7607	.045	.9098	.020
.0183	.005	.1366	.030	.3211	.055	.7683	.044	.9147	.019
.0225	.006	.1422	.031	.3326	.056	.7756	.043	.9195	.018
.0267	.007	.1478	.032	.3450	.057	.7828	.042	.9242	.017
.0309	.008	.1535	.033	.3585	.058	.7898	.041	.9289	.016
.0352	.009	.1594	.034	.3735	.059	.7966	.040	.9335	.015
.0395	.010	.1653	.035	.3904	.060	.8033	.039	.9381	.014
.0439	.011	.1713	.036	.4105	.061	.8098	.038	.9427	.013
.0483	.012	.1775	.037	.4367	.062	.8162	.037	.9472	.012
.0527	.013	.1837	.038	.5632	.061	.8224	.036	.9516	.011
.0572	.014	.1901	.039	.5894	.060	.8286	.035	.9560	.010
.0618	.015	.1966	.040	.6095	.059	.8346	.034	.9604	.009
.0664	.016	.2033	.041	.6264	.058	.8405	.033	.9647	.008
.0710	.017	.2101	.042	.6414	.057	.8464	.032	.9690	.007
.0757	.018	.2171	.043	.6549	.056	.8521	.031	.9732	.006
.0804	.019	.2243	.044	.6673	.055	.8577	.030	.9774	.005
.0852	.020	.2316	.045	.6788	.054	.8633	.029	.9816	.004
.0901	.021	.2392	.046	.6897	.053	.8687	.028	.9857	.003
.0950	.022	.2470	.047	.7000	.052	.8741	.027	.9898	.002
.1000	.023	.2550	.048	.7097	.051	.8794	.026	.9939	.001
.1050	.024	.2633	.049	.7190	.050	.8847	.025	0.9979	.000
0.1101		0.2719		0.7280		0.8898		1.0000	

Table 204d. Bessel's Coefficient *B*₂. In critical cases ascend. *B*₂ is always negative.

205. Interpolation Tables

A number of frequently used navigation tables are provided with auxiliary tables to assist in interpolation. Table 1 (Logarithms of Numbers) provides columns of "d" (difference between consecutive entries) and auxiliary "proportional parts" tables. The auxiliary table for the applicable difference "d" is selected and entered with the

digit of the additional place in the entering argument. The value taken from the auxiliary table is *added* to the base value for the next *smaller* number from the main table. Suppose the logarithm (mantissa) for 32747 is desired. The base value for 3274 is 51508, and "d" is 13. The auxiliary table for 13 is entered with 7, and the correction is found to be 9. If this is added to 51508, the interpolated value is found to be 51517. This is the same result that would be

obtained by subtracting 51508 from 51521 (the logarithm for 3275) to obtain 13, multiplying this by 0.7, and adding the result (9) to 51508.

Tables 1 (Logarithms of Numbers) and Table 2 (Natural Trigonometric Functions) provide the difference between consecutive entries, but no proportional parts tables.

The *Nautical Almanac* "Increments and Corrections" are interpolation tables for the hourly entries of Greenwich Hour Angle (GHA) and declination. The increments are the products of the constant value used as the change of GHA in 1 hour and the fractional part of the hour. The corrections provide for the difference between the actual change of GHA in 1 hour and the constant value used. The corrections also provide the product of the change in declination in 1 hour and the fractional part of the hour.

The main part of the four-page interpolation table of *Pub. No. 229* is basically a multiplication table providing tabulations of:

$$\text{Altitude Difference} \times \frac{\text{Declination Increment}}{60'}$$

The design of the table is such that the desired product must be derived from component parts of the altitude difference. The first part is a multiple of 10' (10', 20', 30', 40', or 50') of the altitude difference; the second part is the remainder in the range 0.0' to 9.9'. For example, the component parts of altitude difference 44.3' are 40' and 4.3'.

In the use of the first part of the altitude difference, the table arguments are declination increment (Dec. Inc.) and the integral multiple of 10' in the altitude difference, d. As shown in Figure 205a, the respondent is:

$$\text{Tens} \times \frac{\text{Dec. Inc.}}{60'}$$

In the use of the second part of the altitude difference, the interpolation table arguments are the nearest Dec. Inc. ending in 0.5' and Units and Decimals. The respondent is:

$$\text{Units and Decimals} \times \frac{\text{Dec. Inc.}}{60'}$$

In computing the table, the values in the Tens part of the multiplication table were modified by small quantities varying from -0.042' to +0.033' before rounding to the tabular precision to compensate for any difference between the actual Dec. Inc. and the nearest Dec. Inc. ending in 0.5' when using the Units and Decimals part of the table.

Using the interpolation table shown in Figure 205b to obtain the altitude for 51°30' from the data of Table 204b (*Data from Pub. No. 229*), the linear correction for the first difference (+0.5') is +0.3'. This correction is extracted from the Units and Decimals block opposite the Dec. Inc. (30.0').

Dec. Inc.	Altitude Difference (d)										Double Second Diff. and Corr.					
	Tens					Units										
	10'	20'	30'	40'	50'	0'	1'	2'	3'	4'		5'	6'	7'	8'	9'
45.0					.0											18.1
45.1					.1											20.3
45.2					.2											22.4
45.3					.3											24.5
45.4					.4											26.7
45.5	7.6	15.2	22.8	30.3	37.9	0.2	1.0	1.7	2.5	3.3	4.0	4.8	5.5	6.3	7.1	28.8
45.6					.5											30.9
45.7					.6											33.1
45.8					.7											35.2
45.9					.8											
					.9											

Figure 205a. Interpolation table.

The correction for the **double second difference (DSD)** is extracted from the DSD subtable opposite the block in which the Dec. Inc. is found. The argument for entering this critical table is the DSD (-4.4'). The DSD correction is +0.3'. Therefore,

$$\begin{aligned} H_c &= h_t + \text{first difference correction} + \text{DSD correction} \\ &= 64^\circ 11.0' + 0.3' + 0.3' = 64^\circ 11.6' \end{aligned}$$

Dec. Inc.	Altitude Difference (d)										Double Second Diff. and Corr.						
	Tens					Units											
	10'	20'	30'	40'	50'	0'	1'	2'	3'	4'		5'	6'	7'	8'	9'	
30.0	5.0	10.0	15.0	20.0	25.0	.0	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.6	4.1	4.6	0.8
30.1	5.0	10.0	15.0	20.0	25.1	.1	0.1	0.6	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	2.4
30.2	5.0	10.0	15.1	20.1	25.1	.2	0.1	0.6	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.7	4.0
30.3	5.0	10.1	15.1	20.2	25.2	.3	0.2	0.7	1.2	1.7	2.2	2.7	3.2	3.7	4.2	4.7	5.6
30.4	5.1	10.1	15.2	20.3	25.3	.4	0.2	0.7	1.2	1.7	2.2	2.7	3.3	3.8	4.3	4.8	7.2
30.5	5.1	10.2	15.3	20.3	25.4	.5	0.3	0.8	1.3	1.8	2.3	2.8	3.3	3.8	4.3	4.8	8.8
30.6	5.1	10.2	15.3	20.4	25.5	.6	0.3	0.8	1.3	1.8	2.3	2.8	3.4	3.9	4.4	4.9	10.4
30.7	5.1	10.3	15.4	20.5	25.6	.7	0.4	0.9	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9	12.0
30.8	5.2	10.3	15.4	20.6	25.7	.8	0.4	0.9	1.4	1.9	2.4	2.9	3.5	4.0	4.5	5.0	13.6
30.9	5.2	10.3	15.5	20.6	25.8	.9	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	15.2
																	16.8

Figure 205b. Interpolation table.

More on Second Differences using Pub 229. The accuracy of linear interpolation usually decreases as the altitude increases. At altitudes above 60° it may be necessary to include the effect of second differences in the interpolation. When the altitude difference, d, is printed in italic type followed by a small dot, the second-difference correction may exceed 0.25', and should normally be applied. The need for a second-difference correction is illustrated by the graph of Table 205c data in Figure 205d.

LHA 28°, Lat. 15° (Same as Dec.)

Table 205c. Data from Pub. No. 229.

Dec.	ht (Tab. Hc)	First Difference	Second Difference
15°	62°58.4'		
		+2.8'	
16°	63°01.2'		-2.0'
		+0.8'	
17°	63°02.0'		-2.1'
		-1.3'	
18°	63°00.7'		

Table 205c. Data from Pub. No. 229.

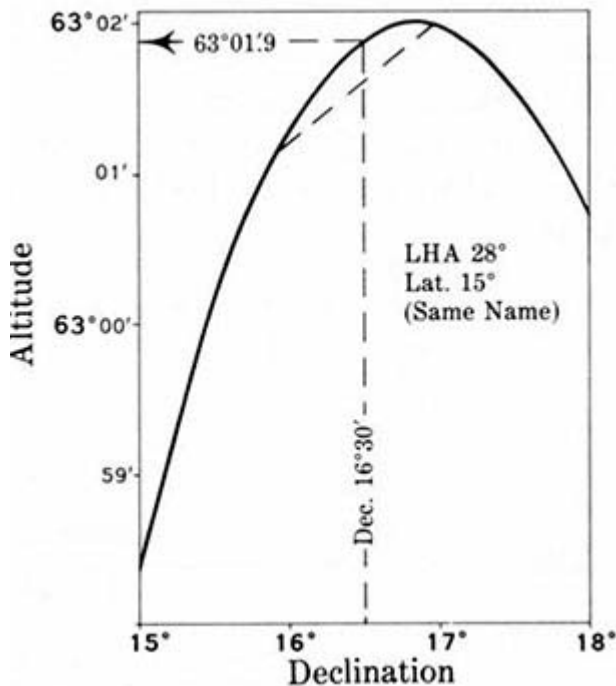


Figure 205d. Graph of Table 205c Data.

Other than graphically, the required correction for the effects of second differences is obtained from the appropriate subtable of the Interpolation Table. However, before the Interpolation Table can be used for this purpose, what is known as the double-second difference (DSD) must be formed.

Forming the Double-Second Difference (DSD). The double-second difference is the sum of two successive second differences. Although second differences are not tabulated, the DSD can be formed readily by subtracting, algebraically, the tabular altitude difference immediately above the respondent altitude difference from the tabular altitude difference immediately below. The result will always be a negative value.

The Double-Second Difference Correction. As shown in Figure 205a, that compartment of the DSD table opposite the block in which the Dec. Inc. is found is entered with the DSD to obtain the DSD correction to the altitude. The correction is always plus. Therefore, the sign of the DSD need not be recorded. When the DSD entry corresponds to an exact tabular value, always use the upper of the two possible corrections.

Example of the Use of the Double-Second Difference. As an example of the use of the double-second difference (DSD) the computed altitude and true azimuth are determined for Lat. 15°N, LHA 28°, and Dec. 16°30.0'N. Data are exhibited in Figure 205a.

The respondents for the entering arguments (Lat. 15° Same Name as Declination, LHA 28°, and Dec. 16°) are:

tabular altitude,	ht	63°01.2'
altitude difference,	d	(+)0.8'
azimuth angle,	Z	84.1°

Table 205e

The linear interpolation correction to the tabular altitude for Dec. Inc. 30.0' is (+)0.4'.

$$Hc = ht + \text{linear correction} + \text{DSD correction}$$

However, by inspection of Figure 205d, illustrating this solution graphically, the computed altitude should be 63°01.9'. The actual change in altitude with an increase in declination is nonlinear. The altitude value lies on the curve between the points for declination 16° and declination 17° instead of the straight line connecting these points.

The DSD is formed by subtracting, algebraically, the tabular altitude difference immediately above the respondent altitude difference from the tabular altitude difference immediately below. Thus, the DSD is formed by algebraically subtracting (+)2.8' from (-)1.3'; the result is (-)4.1'.

As shown in Figure 205f, that compartment of the DSD table opposite the block in which the Dec. Inc. (30.0') is found is entered with the DSD (4.1') to obtain the DSD correction to the altitude. The correction is 0.3'. The correction is always plus.

$$Hc = ht + \text{linear correction} + \text{DSD correction}$$

$$Hc = 63°01.2' + 0.4' + 0.3' = 63°01.9'$$

Extrapolation.-The extending of a table is usually performed by assuming that the difference between the last few tabulated entries will continue at the same rate. This assumption is strictly correct only if the change is truly linear, but in most tables the assumption provides satisfactory results for a *slight* extension beyond tabulated values. The extent to which the assumption can be used reliably can of-

28°, 332° L.H.A.

Dec.	LATITUDE SAME NAME								
	15°			16°			17°		
	Hc	d	Z	Hc	d	Z	Hc	d	Z
9	61 59.3	+14.7	99.1	61 48.8	+16.8	101.0	61 36.4	+18.9	102.8
10	62 14.0	+12.8	97.1	62 05.6	+15.0	98.9	61 55.3	+17.1	100.8
11	62 26.8	+10.9	95.0	62 20.6	+13.1	96.9	62 12.4	+15.3	98.8
12	62 37.7	+8.9	92.8	62 33.7	+11.1	94.8	62 27.7	+13.4	96.7
13	62 46.6	+6.9	90.7	62 44.8	+9.2	92.6	62 41.1	+11.4	94.6
14	62 53.5	+4.9	88.5	62 54.0	+7.2	90.5	62 52.5	+9.5	92.4
15	62 58.4	+2.8	86.3	63 01.2	+5.1	88.3	63 02.0	+7.4	90.2
16	63 01.2	+0.8	84.1	63 06.3	+3.1	86.1	63 09.4	+5.4	88.0
17	63 02.0	-7.3	81.9	63 09.4	+1.0	83.9	63 14.8	+3.3	85.8
18	63 00.7	-3.3	79.7	63 10.4	-1.0	81.6	63 18.1	+1.3	83.6
19	62 57.4	-5.4	77.5	63 09.4	-3.1	79.4	63 19.4	-0.9	81.4

Data from Page 58 of Pub 229 - Volume 2

Dec. Inc.	INTERPOLATION TABLE																		Double Second Diff. and Corr.
	Altitude Difference (d)																		
	Tens					Decimals					Units								
	10'	20'	30'	40'	50'	0'	1'	2'	3'	4'	5'	6'	7'	8'	9'				
30.0	5.0	10.0	15.0	20.0	25.0	.0	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.6	4.1	4.6	0.8	0.1	
30.1	5.0	10.0	15.0	20.0	25.1	.1	0.1	0.6	1.1	1.6	2.1	2.6	3.1	3.6	4.1	4.6	2.4	0.2	
30.2	5.0	10.0	15.1	20.1	25.1	.2	0.1	0.6	1.1	1.6	2.1	2.6	3.2	3.7	4.2	4.7	4.0	0.3	
30.3	5.0	10.1	15.1	20.2	25.2	.3	0.2	0.7	1.2	1.7	2.2	2.7	3.2	3.7	4.2	4.7	5.6	0.3	
30.4	5.1	10.1	15.2	20.3	25.3	.4	0.2	0.7	1.2	1.7	2.2	2.7	3.3	3.8	4.3	4.8	7.2	0.4	
30.5	5.1	10.2	15.3	20.3	25.4	.5	0.3	0.8	1.3	1.8	2.3	2.8	3.3	3.8	4.3	4.8	8.8	0.5	
30.6	5.1	10.2	15.3	20.4	25.5	.6	0.3	0.8	1.3	1.8	2.3	2.8	3.4	3.9	4.4	4.9	10.4	0.7	
30.7	5.1	10.3	15.4	20.5	25.6	.7	0.4	0.9	1.4	1.9	2.4	2.9	3.4	3.9	4.4	4.9	12.0	0.8	
30.8	5.2	10.3	15.4	20.6	25.7	.8	0.4	0.9	1.4	1.9	2.4	2.9	3.5	4.0	4.5	5.0	13.6	0.9	
30.9	5.2	10.3	15.5	20.6	25.8	.9	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0	15.2	1.0	
																	16.8		

Data from Interpolation Table

Figure 205f. Interpolation blocks from Pub No. 229.

ten be determined by noting the last few differences. If the "second differences" (differences between consecutive differences) are nearly zero, the curve is nearly a straight line, for a short distance. But if consecutive second differences are appreciable, extrapolation is not reliable. For examples of linear and nonlinear relationships, refer to the first page of Table 3 (Common Logarithms of Trigonometric Functions) and compare the tabulated differences of the logarithms of secant (approximately linear on this page) and sine (nonlinear on this page).

As an example of extrapolation, consider Table 22 (Amplitudes). Suppose the amplitude for latitude 45°, declination 24.3° is desired. The last declination entry is 24.0°. The amplitude for declination 23.5° is 34.3°, and for declination 24.0° it is 35.1°. The difference is (+) 0.8°. Assuming this same difference between declinations 24.0° and 24.5°, one finds the value for 24.3° is $35.1 + \left(\frac{3}{5} \cdot 0.8\right) = 35.6$. Below latitude 50° this table is so nearly linear that extrapolation can be carried to declination 30° without serious error.

For double or triple extrapolation, differences are found as in single interpolation.

206. General Comments

As a general rule, the final answer should not be given to greater precision than tabulated values. A notable exception to this rule is the case where tabulated values are known to be exact, as in Table 201b. A slight increase in accuracy can sometimes be attained by retaining one additional place in the solution until the final answer. Suppose, for instance, that the corrections for triple interpolation are (+)0.2, (+)0.3, and (-)0.3. The total correction is (+)0.2. If the total correction, rounded to tenths, had been obtained from the sum of (+)0.17, (+)0.26, and (-)0.34, the

correct total would have been (+)0.09= (+)0.1. The retaining of one additional place may be critical if the correction factors end in 0.5. Thus, in double interpolation, one correction value might be (+)0.15, and the other (-)0.25. The correct total is (-)0.1. But if the individual differences are rounded to (+)0.2 and (-)0.2, the total is 0.0.

The difference used for establishing the proportion is also a matter subject to some judgment. Thus, if the latitude is 17°14.6', it might be rounded to 17.2° for many purposes. Slightly more accurate results can sometimes be obtained

by retaining the minutes, using $\frac{14.6}{60}$ instead of 0.2. If the difference to be multiplied by this proportion is small, the increase in accuracy gained by using the more exact value is small, but if the difference is large, the gain might be considerable. Thus, if the difference is 0.2°, the correction by using either $\frac{14.6}{60}$ or 0.2 is less than 0.05°, or 0.0° to the

nearest 0.1°. But if the difference is 3.2°, the value by $\frac{14.6}{60}$ is 0.8°, and the value by 0.2 is 0.6°.

If the tabulated entries involved in an interpolation are all positive or all negative, the interpolation can be carried out on either a numerical or an algebraic basis. Most navigators prefer the former, carrying out the interpolation as if all entries were positive, and giving to the interpolated value the common sign of all entries. When both positive and negative entries are involved, all differences and corrections should be on an algebraic basis, and careful attention should be given to signs. Thus, if single interpolation is to be performed between values of (+)0.9 and (-)0.4, the difference is $0.9 - (-0.4) = 0.9 + 0.4 = 1.3$. If the correction is 0.2 of this difference, it is (-)0.3 if applied to (+)0.9, and (+)0.3 if applied to (-)0.4. In the first case, the interpolated value is $(+)0.9 - 0.3 = (+)0.6$. In the second case, it is $(-)0.4 + 0.3 = (-)0.1$. If the correction had been 0.4 of the

difference, it would have been $(-)0.5$ in the first case, and $(+)0.5$ in the second. The interpolated value would have been $(+)0.9 - 0.5 = (+)0.4$, or $(-)0.4 + 0.5 = (+)0.1$, respectively.

Because of the variety in methods of interpolation used, solutions by different persons may differ slightly.