# EXPLANATION OF NAVIGATION TABLES 

## Mathematical Tables

Table 1. Logarithms of Numbers - The first page of this table gives the complete common logarithm (characteristic and mantissa) of numbers 1 through 250. Succeeding pages give the mantissa only of the common logarithm of any number. Values are given for four significant digits of entering values, the first three being in the left-hand column, and the fourth at the heading of one of the other columns. Thus, the mantissa of a threedigit number is given in the column headed 0 , on the line with the given number; while the mantissa of a four-digit number is given in the column headed by the fourth digit, on the line with the first three digits. As an example, the mantissa of 328 is 51587 , while that of 3.284 is 51640 . For additional digits, interpolation should be used. The difference between each tabulated mantissa and the next larger tabulated mantissa is given in the "d" column to the right of the smaller mantissa. This difference can be used to enter the appropriate proportional parts ("Prop. parts") auxiliary table to interpolate for the fifth digit of the given number. If an accuracy of more than five significant digits is to be preserved in a computation, a table of logarithms to additional decimal places should be used. For a number of one or two digits, use the first page of the table or add zeros to make three digits. That is, the mantissa of 3,30 , and 300 is the same, 47712. Interpolation on the first page of the table is not recommended. The second part should be used for values not listed on the first page.

Table 2. Natural Trigonometric Functions - This table gives the values of natural sines, cosecants, tangents, cotangents, secants, and cosines of angles from $0^{\circ}$ to $180^{\circ}$, at intervals of $1^{\prime}$. For angles between $0^{\circ}$ and $45^{\circ}$ use the column labels at the top and the minutes at the left; for angles between $45^{\circ}$ and $90^{\circ}$ use the column labels at the bottom and the minutes at the right; for angles between $90^{\circ}$ and $135^{\circ}$ use the column labels at the bottom and the minutes at the left; and for angles between $135^{\circ}$ and $180^{\circ}$ use the column labels at the top and the minutes at the right. These combinations are indicated by the arrows accompanying the figures representing the number of degrees. For angles between $180^{\circ}$ and $360^{\circ}$, subtract $180^{\circ}$ and proceed as indicated above to obtain the numerical values of the various functions.

Differences between consecutive entries are shown in the "Diff. 1"" column to the right of each column of values of a trigonometric function, as an aid to interpolation. These differences are one-half line out of step with the numbers to which they apply, as in a critical table. Each difference ap-
plies to the values half a line above and half a line below. To determine the correction to apply to the value for the smaller entering angle, multiply the difference by the number of tenths of a minute (or seconds $\div 60$ ) of the entering angle. Note whether the function is increasing or decreasing, and add or subtract the correction as appropriate, so that the interpolated value lies between the two values between which interpolation is made.

Table 3. Logarithms of Trigonometric Functions This table gives the common logarithms (+10) of sines, cosecants, tangents, cotangents, secants, and cosines of angles from $0^{\circ}$ to $180^{\circ}$, at intervals of $1^{\prime}$. For angles between $0^{\circ}$ and $45^{\circ}$ use the column labels at the top and the minutes at the left; for angles between $45^{\circ}$ and $90^{\circ}$ use the column labels at the bottom and the minutes at the right; for angles between $90^{\circ}$ and $135^{\circ}$ use the column labels at the bottom and the minutes at the left; and for angles between $135^{\circ}$ and $180^{\circ}$ use the column labels at the top and the minutes at the right. These combinations are indicated by the arrows accompanying the figures representing the number of degrees. For angles between $180^{\circ}$ and $360^{\circ}$, subtract $180^{\circ}$ and proceed as indicated above to obtain the numerical values of the various functions.

Differences between consecutive entries are shown in the "Diff. 1"" columns, except that one difference column is used for both sines and cosecants, another for both tangents and cotangents, and a third for both secants and cosines. These differences, given as an aid to interpolation, are onehalf line out of step with the numbers to which they apply, as in a critical table. Each difference applies to the values half a line above and half a line below. To determine the correction to apply to the value for the smaller entering angle, multiply the difference by the number of tenths of a minute (or seconds $\div 60$ ) of the entering angle. Note whether the function is increasing or decreasing, and add or subtract the correction as appropriate, so that the interpolated value lies between the two values between which interpolation is made.

Table 4. Traverse Table - This table can be used in the solution of any of the sailings except great-circle and composite. In providing the values of the difference of latitude and departure corresponding to distances up to 600 miles and for courses for every degree of the compass, Table 4 is essentially a tabulation of the solutions of plane right triangles. Since the solutions are for integral values of the acute angle and the distance, interpolation for intermediate values may be required. Through appropriate interchanges of the headings of the
columns, solutions for other than plane sailings can be made. The interchanges of the headings of the different columns are summarized at the foot of each table opening.

The distance, difference of latitude, and departure columns are labeled Dist., D. Lat., and Dep., respectively.

For solution of a plane right triangle, any number N in the distance column is the hypotenuse; the number opposite in the difference of latitude column is N times the cosine of the acute angle; and the other number opposite in the departure column is N times the sine of the acute angle. Or, the number in the column labeled D. Lat. is the value of the side adjacent and the number in the column labeled Dep. is the value of the side opposite the acute angle.

Appendix B. Haversines - These tables list the common logarithms ( +10 ) of haversines and natural haversines of angles from $0^{\circ}$ to $360^{\circ}$, at intervals of $1^{\prime}$. For angles between $0^{\circ}$ and $180^{\circ}$ use the degrees as given at the tops of the columns and the minutes at the left; for angles between $180^{\circ}$ and $360^{\circ}$ use the degrees as given at the bottom of the columns and the minutes at the right.

## Cartographic Tables

Table 5. Natural and Numerical Chart Scales This table gives the numerical scale equivalents for various natural or fractional chart scales. The scale of a chart is the ratio of a given distance on the chart to the actual distance which it represents on the earth. The scale may be expressed as a simple ratio or fraction, known as the natural scale. For example, 1:80,000 or $\frac{1}{80000}$ means that one unit (such as an inch) on the chart represents 80,000 of the same unit on the surface of the earth. The scale may also be expressed as a statement of that distance on the earth shown as one unit (usually an inch) on the chart, or vice versa. This is the numerical scale.

The table was computed using 72,913.39 inches per nautical mile and 63,360 inches per statute mile.

Table 6. Meridional Parts - In this table the meridional parts used in the construction of Mercator charts and in Mercator sailing are tabulated to one decimal place for each minute of latitude from the equator to the poles.

The table was computed using the formula:
$M=a \log _{e} 10 \log \tan \left(45+\frac{L}{2}\right)-a\left(e^{2} \sin L+\frac{e^{4}}{3} \sin ^{3} L+\right.$ $\left.\frac{e^{6}}{5} \sin ^{5} L+\ldots\right)$,
in which M is the number of meridional parts between the equator and the given latitude, a is the equatorial radius of the earth, expressed in minutes of arc of the equator, or
$\mathrm{a}=\frac{21600}{2 \pi}=3437.74677078(\log =3.5362739)$,
$\log _{\mathrm{e}}$ is the natural (Naperian) logarithm, using the base $e=2.71828182846$,

$$
\log _{\mathrm{e}} 10=2.3025851 \quad(\log =0.36221569)
$$

L is the latitude,
$f$ is earth's flattening, or

$$
\begin{aligned}
& f=\frac{1}{298.257223563} \\
& =3.35281066475 \cdot 10^{-3} \quad(\log =7.474591-10)
\end{aligned}
$$

the squared eccentricity of the earth, $e^{2}$ [not to be confused with Euler's constant, the base of natural logarithms] is

$$
\begin{aligned}
& e^{2}=2 f-f^{2} \\
& =6.694379990141 \cdot 10^{-3} \quad(\log =7.1742896-10)
\end{aligned}
$$

Using these values,

$$
\mathrm{a} \log _{\mathrm{e}} 10=7915.7(\log =3.8984893)
$$

$$
\mathrm{ae}^{2}=23.01358319(\log =1.3619842)
$$

$$
\frac{\mathrm{ae}^{4}}{3}=0.05135389(\log =8.2894349-10)
$$

$$
\frac{\mathrm{ae}^{6}}{5}=0.00020627(\log =6.6855639-10)
$$

Hence, the formula becomes

$$
\begin{aligned}
& \mathrm{M}=7915.7 \log \tan \left(45^{\circ}+\frac{\mathrm{L}}{2}\right)-23.01358319 \\
& \sin \mathrm{~L}-0.05135389 \sin ^{3} \mathrm{~L}-0.00020627 \sin ^{5} \mathrm{~L} \ldots
\end{aligned}
$$

The constants used in this derivation and in the table are based upon the World Geodetic System 1984 (WGS 84) ellipsoid.

Table 7. Length of a Degree of Latitude and Longitude - This table gives the length of one degree of latitude and longitude at intervals of $1^{\circ}$ from the equator to the poles. In the case of latitude, the values given are the lengths of the arcs extending half a degree on each side of
the tabulated latitudes. Lengths are given in nautical miles, statute miles, feet, and meters.

The values were computed in meters, using parameters of the World Geodetic System 1984 (WGS 84) ellipsoid, and converted to other units. The following formulas were used:
$M=\left(\frac{\pi}{180}\right)\left[\frac{a\left(1-e^{2}\right)}{w^{3}}\right]$ where
$a=6378137$, the semi - major axis of the WGS 84 ellipsoid

$$
\begin{aligned}
& \mathrm{e}^{2}=2 \mathrm{f}-\mathrm{f}^{2}=6.694379990141 \cdot 10^{-3} \\
& w=\sqrt{1-e^{2} \sin ^{2} \varphi} \\
& \varphi=\text { geodetic latitude }
\end{aligned}
$$

And

$$
P=\left(\frac{\pi}{180}\right)\left[\frac{a(\cos \varphi)}{w}\right]
$$

## Piloting Tables

Table 8. Conversion Table for Meters, Feet, and Fathoms - The number of feet and fathoms corresponding to a given number of meters, and vice versa, can be taken directly from this table for any value of the entering argument from 1 to 120 . The entering value can be multiplied by any power of 10 , including negative powers, if the corresponding values of the other units are multiplied by the same power. Thus, 420 meters are equivalent to 1378.0 feet, and 11.2 fathoms are equivalent to 20.483 meters.

The table was computed by means of the relationships:
1 meter $=39.370079$ inches,
1 foot $=12$ inches,
1 fathom $=6$ feet.
Table 9. Conversion Table for Nautical and Statute Miles - This table gives the number of statute miles corresponding to any whole number of nautical miles from 1 to 100, and the number of nautical miles corresponding to any whole number of statute miles within the same range. The entering value can be multiplied by any power of 10 , including negative powers, if the corresponding value of the other unit is multiplied by the same power. Thus, 2,700 nautical miles are equivalent to $3,107.1$ statute miles, and 0.3 statute mile is equivalent to 0.2607 nautical mile.

The table was computed using the conversion factors:

1 nautical mile $=1.15077945$ statute miles,
1 statute mile $=0.86897624$ nautical mile .

Table 10. Speed Table for Measured Mile - To find the speed of a vessel on a measured nautical mile in a given number of minutes and seconds of time, enter this table at the top or bottom with the number of minutes, and at either side with the number of seconds. The number taken from the table is speed in knots. Accurate results can be obtained by interpolating to the nearest 0.1 second.

This table was computed by means of the formula:
$S=\frac{3600}{T}$, in which $S$ is speed in knots, and $T$ is elapsed time in seconds.

Table 11. Speed, Time, and Distance Table - To find the distance steamed at any given speed between 0.5 and 40 knots in any given number of minutes from 1 to 60 , enter this table at the top with the speed, and at the left with the number of minutes. The number taken from the table is the distance in nautical miles. If hours are substituted for minutes, the tabulated distance should be multiplied by 60 ; if seconds are substituted for minutes, the tabulated distance should be divided by 60 .

The table was computed by means of the formula:
$\mathrm{D}=\frac{\mathrm{ST}}{60}$, in which D is distance in nautical miles,
S is speed in knots, and T is elapsed time in minutes.

Table 12. Distance of the Horizon - This table gives the distance in nautical and statute miles of the visible sea horizon for various heights of eye in feet and meters. The actual distance varies somewhat as refraction changes. However, the error is generally less than that introduced by nonstandard atmospheric conditions. Also the formula used contains an approximation which introduces a small error at the greatest heights tabulated.
The table was computed using the formula:

$$
\mathrm{D}=\sqrt{\frac{2 \mathrm{r}_{\mathrm{o}} \mathrm{~h}_{\mathrm{f}}}{6076.1 \beta_{\mathrm{o}}}}
$$

in which D is the distance to the horizon in nautical miles; ro is the mean radius of the earth, 3440.1 nautical miles; $\mathrm{h}_{\mathrm{f}}$ is the height of eye in feet; and $\beta_{o}(0.8279)$ accounts for terrestrial refraction.

This formula simplifies to: $D(n m)=1.169 \sqrt{h_{f}}$

$$
(\text { statute miles })=1.345 \sqrt{\mathrm{~h}_{\mathrm{f}}}
$$

Table 13. Geographic Range - This table gives the geographic range or the maximum distance at which the curvature of the earth permits a light to be seen from a particular height of eye without regard to the luminous intensity of the light. The geographic range depends upon the height of both the light and the eye of the observer.

The table was computed using the formula:

$$
\mathrm{D}=1.17 \sqrt{\mathrm{H}}+1.17 \sqrt{\mathrm{~h}}
$$

in which D is the geographic range in nautical miles, H is the height in feet of the light above sea level, and $h$ is the height in feet of the eye of the observer above sea level.

Table 14. Dip of the Sea Short of the Horizon - If land, another vessel, or other obstruction is between the observer and the sea horizon, use the waterline of the obstruction as the horizontal reference for altitude measurements, and substitute dip from this table for the dip of the horizon (height of eye correction) given in the Nautical Almanac. The values below the bold rules are for normal dip, the visible horizon being between the observer and the obstruction.

The table was computed with the formula:

$$
\mathrm{D}_{\mathrm{s}}=60 \tan ^{-1}\left(\frac{\mathrm{~h}_{\mathrm{f}}}{6076.1 \mathrm{~d}_{\mathrm{s}}}+\frac{\beta_{\mathrm{o}} \mathrm{~d}_{\mathrm{s}}}{2 \mathrm{r}_{\mathrm{o}}}\right)
$$

in which $\mathrm{D}_{\mathrm{s}}$ is the dip short of the sea horizon, in minutes of $\operatorname{arc} ; \mathrm{h}_{\mathrm{f}}$ is the height of eye of the observer above sea level in feet; $\beta_{0}(0.8321)$ accounts for terrestrial refraction; $r_{0}$ is the mean radius of the earth, 3440.1 nautical miles; and $d_{s}$ is the distance to the waterline of the obstruction in nautical miles.

Table 15. Distance by Vertical Angle Measured Between Sea Horizon and Top of Object Beyond Sea Horizon - This table tabulates the distance to an object of known height above sea level when the object lies beyond the horizon. The vertical angle between the top of the object and the visible horizon is measured with a sextant and corrected for index error and dip only. The table is entered with the difference in the height of the object and the height of eye of the observer and the corrected vertical angle; and the distance in nautical miles is taken directly from the table. An error may be introduced if refraction differs from the standard value used in the computation of the table.

The table was computed using the formula:

$$
\mathrm{D}=\sqrt{\left(\frac{\tan \alpha}{0.0002419}\right)^{2}+\frac{\mathrm{H}-\mathrm{h}}{0.7349}}-\frac{\tan \alpha}{0.0002419}
$$

in which D is the distance in nautical miles, $\alpha$ is the corrected vertical angle, $H$ is the height of the top of the object above sea level in feet, and $h$ is the height of eye of the observer above sea level in feet. The constants 0.0002419 and
0.7349 account for terrestrial refraction.

Table 16. Distance by Vertical Angle Measured Between Waterline at Object and Top of Object - This table tabulates the angle subtended by an object of known height lying at a particular distance within the observer's visible horizon or vice versa.

The table provides the solution of a plane right triangle having its right angle at the base of the observed object and its altitude coincident with the vertical dimension of the observed object. The solutions are based upon the following simplifying assumptions: (1) the eye of the observer is at sea level, (2) the sea surface between the observer and the object is flat, (3) atmospheric refraction is negligible, and (4) the waterline at the object is vertically below the peak of the object. The error due to the height of eye of the observer does not exceed 3 percent of the distance-off for sextant angles less than $20^{\circ}$ and heights of eye less than one-third of the object height. The error due to the waterline not being below the peak of the object does not exceed 3 percent of the distance-off when the height of eye is less than one-third of the object height and the offset of the waterline from the base of the object is less than one-tenth of the distance-off. Errors due to earth's curvature and atmospheric refraction are negligible for cases of practical interest.

Table 17. Distance by Vertical Angle Measured Between Waterline at Object and Sea Horizon Beyond Object - This table tabulates the distance to an object lying within or short of the horizon when the height of eye of the observer is known. The vertical angle between the waterline at the object and the visible (sea) horizon beyond is measured and corrected for index error. The table is entered with the corrected vertical angle and the height of eye of the observer in feet; the distance in yards is taken directly from the table

The table was computed from the formula:

$$
\begin{aligned}
& \tan h_{s}=(A-B) \div(1+A B) \text { where } \\
& A=\frac{h}{d_{s}}+\frac{\beta_{\mathrm{o}} d_{s}}{2 r_{\mathrm{o}}} \text { and } \\
& B=\sqrt{2 \beta_{\mathrm{o}} \mathrm{~h} / \mathrm{r}_{\mathrm{o}}}
\end{aligned}
$$

in which $\beta_{0}(0.8279)$ accounts for terrestrial refraction, $r_{0}$ is the mean radius of the earth, 3440.1 nautical miles; $h$ is the height of eye of the observer in feet; $h_{\mathrm{s}}$ is the observed vertical angle corrected for index error; and $\mathrm{d}_{\mathrm{s}}$ is the distance to the waterline of the object in nautical miles.

Table 18. Distance of an Object by Two Bearings To determine the distance of an object as a vessel on a steady course passes it, observe the difference between the course and two bearings of the object, and note the time interval between bearings. Enter this table with the two
differences. Multiply the distance run between bearings by the number in the first column to find the distance of the object at the time of the second bearing, and by the number in the second column to find the distance when abeam.

The table was computed by solving plane oblique and right triangles.

## Celestial Navigation Tables

Table 19. Table of Offsets - This table gives the corrections to the straight line of position (LOP) as drawn on a chart or plotting sheet to provide a closer approximation to the arc of the circle of equal altitude, a small circle of radius equal to the zenith distance.

In adjusting the straight LOP to obtain a closer approximation of the arc of the circle of equal altitude, points on the LOP are offset at right angles to the LOP in the direction of the celestial body. The arguments for entering the table are the distance from the DR to the foot of the perpendicular and the altitude of the body.

The table was computed using the formulas:

$$
\begin{aligned}
& R=3438^{\prime} \cot h \\
& \sin \theta=D / R \\
& X=R(1-\cos \theta)
\end{aligned}
$$

in which X is the offset, R is the radius of a circle of equal altitude for altitude $h$, and D is the distance from the intercept to the point on the LOP to be offset.

Table 20. Meridian Angle and Altitude of a Body on the Prime Vertical Circle - A celestial body having a declination of contrary name to the latitude does not cross the prime vertical above the celestial horizon, its nearest approach being at rising or setting.

If the declination and latitude are of the same name, and the declination is numerically greater, the body does not cross the prime vertical, but makes its nearest approach (in azimuth) when its meridian angle, east or west, and altitude are as shown in this table, these values being given in italics above the heavy line. At this time the body is stationary in azimuth.

If the declination and latitude are of the same name and numerically equal, the body passes through the zenith as it crosses both the celestial meridian and the prime vertical, as shown in the table.

If the declination and latitude are of the same name, and the declination is numerically less, the body crosses the prime vertical when its meridian angle, east or west, and altitude are as tabulated in vertical type below the heavy line.

The table is entered with declination of the celestial body and the latitude of the observer. Computed altitudes are given, with no allowance made for refraction, dip, parallax, etc. The tabulated values apply to any celestial body, but values are not given for declination greater than $23^{\circ}$ because the tabulated information is generally desired for the sun only.

The table was computed using the following formulas, derived by Napier's rules:

Nearest approach (in azimuth) to the prime vertical:

$$
\csc \mathrm{h}=\sin \mathrm{d} \csc \mathrm{~L}
$$

$\sec \mathrm{t}=\tan \mathrm{d} \cot \mathrm{L}$

On the prime vertical:

```
sin h = sind cscL
```

$\cos \mathrm{t}=\tan \mathrm{d} \cot \mathrm{L}$

In these formulas, $h$ is the altitude, d is the declination, L is the latitude, t is the meridian angle.

Table 21. Latitude and Longitude Factors - The latitude obtained by an ex-meridian sight is inaccurate if the longitude used in determining the meridian angle is incorrect. Similarly, the longitude obtained by solution of a time sight is inaccurate if the latitude used in the solution is incorrect, unless the celestial body is on the prime vertical. This table gives the errors resulting from unit errors in the assumed values used in the computations. There are two columns for each tabulated value of latitude. The first gives the latitude factor, f , which is the error in minutes of latitude for a one-minute error of longitude. The second gives the longitude factor, F , which is the error in minutes of longitude for a one-minute error of latitude. In each case, the total error is the factor multiplied by the number of minutes error in the assumed value. Although the factors were originally intended for use in correcting ex-meridian altitudes and time-sight longitudes, they have other uses as well.

The azimuth angle used for entering the table can be measured from either the north or south, through $90^{\circ}$; or it may be measured from the elevated pole, through $180^{\circ}$. If the celestial body is in the southeast $\left(090^{\circ}-180^{\circ}\right)$ or northwest $\left(270^{\circ}-360^{\circ}\right)$ quadrant, the f correction is applied to the northward if the correct longitude is east of that used in the solution, and to the southward if the correct longitude is west of that used; while the F correction is applied to the eastward if the correct latitude is north of that used in the solution, and to the westward if the correct latitude is south of that used. If the body is in the northeast $\left(000^{\circ}-090^{\circ}\right)$ or southwest $\left(180^{\circ}-270^{\circ}\right)$ quadrant, the correction is applied in the opposite direction. These rules apply in both north and south latitude.

The table was computed using the formulas:

$$
\begin{aligned}
& \mathrm{f}=\cos \mathrm{L} \tan \mathrm{Z}=\frac{1}{\sec \mathrm{~L} \cot \mathrm{Z}}=\frac{1}{\mathrm{~F}} \\
& \mathrm{~F}=\sec \mathrm{L} \cot \mathrm{Z}=\frac{1}{\cos \mathrm{~L} \tan \mathrm{Z}}=\frac{1}{\mathrm{f}}
\end{aligned}
$$

in which f is the tabulated latitude factor, L is the latitude, Z is the azimuth angle, and F is the tabulated longitude factor.

Table 22. Amplitudes - This table lists amplitudes of celestial bodies at rising and setting. Enter with the declination of the body and the latitude of the observer. The value taken from the table is the amplitude when the center of the body is on the celestial horizon. For the sun, this occurs when the lower limb is a little more than half a diameter above the visible horizon. For the moon it occurs when the upper limb is about on the horizon. Use the prefix E if the body is rising, and W if it is setting; use the suffix N or S to agree with the declination of the body. Table 23 can be used with reversed sign to correct the tabulations to the values for the visible horizon.

The table was computed using the following formula, derived by Napier's rules:

$$
\sin A=\sec L \sin d
$$

in which A is the amplitude, L is the latitude of the observer, and dis the declination of the celestial body.

Table 23. Correction of Amplitude Observed on the Visible Horizon - This table contains a correction to be applied to the amplitude observed when the center of a celestial body is on the visible horizon, to obtain the corresponding amplitude when the center of the body is on the celestial horizon. For the sun, a planet, or a star, apply the correction in the direction away from the elevated pole, thus increasing the azimuth angle. For the moon apply half the correction toward the elevated pole. This correction can be applied in the opposite direction to a value taken from Table 22 to find the corresponding amplitude when the center of a celestial body is on the visible horizon. The table was computed for a height of eye of 41 feet. For other heights normally encountered, the error is too small to be of practical significance in ordinary navigation.

The values in the table were determined by computing the azimuth angle when the center of the celestial body is on the visible horizon, converting this to amplitude, and determining the difference between this value and the corresponding value from Table 22. Computation of azimuth angle was made for an altitude of ( - ) $0^{\circ} 42.0^{\prime}$ determined as follows:

Azimuth angle was computed by means of the formula:

$$
\cos Z=\frac{\sin d-\sin h \sin L}{\cosh \cos L}
$$

in which Z is the azimuth angle, d is the declination of the celestial body, h is the altitude $\left(-0^{\circ} 42.0^{\prime}\right)$, and L is the latitude of the observer.

Table 24. Altitude Factors - In one minute of time from meridian transit the altitude of a celestial body changes by the amount shown in this table if the altitude is between $6^{\circ}$ and $86^{\circ}$, the latitude is not more than $60^{\circ}$, and the declination is not more than $63^{\circ}$. The values taken from this table are used to enter Table 25 for solving reduction to the meridian (ex-meridian) problems.

For upper transit, use the left-hand pages if the declination and latitude are of the same name (both north or both south) and the right-hand pages if of contrary name. For lower transit, use the values below the heavy lines on the last three contrary-name pages. When a factor is taken from this part of the table, the correction from table 25 is subtracted from the observed altitude to obtain the corresponding meridian altitude. All other corrections are added.

The table was computed using the formula:
$a=1.9635^{\prime \prime} \cos L \cos d \csc (L \sim d)$
in which a is the change of altitude in one minute from meridian transit (the tabulated value), L is the latitude of the observer, and $d$ is the declination of the celestial body.

This formula can be used to compute values outside the limits of the table, but is not accurate if the altitude is greater than $86^{\circ}$.

Table 25. Change of Altitude in Given Time from Meridian Transit - Enter this table with the altitude factor from table 24 and the meridian angle, in either arc or time units, and take out the difference between the altitude at the given time and the altitude at meridian transit. Enter the table separately with whole numbers and tenths of a, interpolating for t if necessary, and add the two values to obtain the total difference. This total can be applied as a correction to observed altitude to obtain the corresponding meridian altitude, adding for upper transit and subtracting for lower transit.

The table was computed using the formulas:

$$
\mathrm{C}=\frac{\mathrm{at}^{2}}{60}
$$

in which C is the tabulated difference to be used as a correction to observed altitude in minutes of arc; $a$ is the altitude factor from table 24 in seconds of arc; and $t$ is the meridian angle in minutes of time.

This formula should not be used for determining values beyond the limits of the table unless reduced accuracy is acceptable.

Table 26. Time Zones, Descriptions, and Suffixes The zone description and the single letter of the alphabet designating a time zone and sometimes used as a suffix to zone time for all time zones are given in this table.

Table 27. Altitude Correction for Air Temperature - This table provides a correction to be applied to the altitude of a celestial body when the air temperature varies from the $50^{\circ} \mathrm{F}$ used for determining mean refraction with the Nautical Almanac. For maximum accuracy, apply index correction and dip to sextant altitude first, obtaining apparent (rectified) altitude for use in entering this table. Enter the table with altitude and air temperature in degrees Fahrenheit. Apply the correction in accordance with its tabulated sign to altitude.

The table was computed using formula:

$$
\text { Correction }=\mathrm{R}_{\mathrm{m}}\left(1-\frac{510}{460+\mathrm{T}}\right)
$$

in which $\mathrm{R}_{\mathrm{m}}$ is mean refraction and T is temperature in degrees Fahrenheit.

Table 28. Altitude Correction for Atmospheric Pressure - This table provides a correction to be applied to the altitude of a celestial body when the atmospheric pressure varies from the 29.83 inches ( 1010 millibars) used for determining mean refraction using the Nautical Almanac. For most accurate results, apply index correction and dip to sextant altitude first, obtaining apparent (rectified) altitude for use in entering this table. Enter the table with altitude and atmospheric pressure. Apply the correction to altitude, adding if the pressure is less than 29.83 inches and subtracting if it is more than 29.83 inches. The table was computed by means of the formula:

$$
\text { Correction }=\mathrm{R}_{\mathrm{m}}\left(1-\frac{\mathrm{P}}{29.83}\right)
$$

in which $R_{m}$ is mean refraction and $P$ is atmospheric pressure in inches of mercury.

## Meteorological Tables

Table 29. Conversion Table for Thermometer Scales - Enter this table with temperature Fahrenheit, F; Celsius (centigrade), C; or Kelvin, K; and take out the corresponding readings on the other two temperature scales.

On the Fahrenheit scale, the freezing temperature of pure water at standard sea level pressure is $32^{\circ}$, and the boiling point under the same conditions is $212^{\circ}$. The corresponding temperatures are $0^{\circ}$ and $100^{\circ}$ on the Celsius scale and $273.15^{\circ}$ and $373.15^{\circ}$, respectively, on the Kelvin scale. The value of (-) $273.15^{\circ} \mathrm{C}$ for absolute zero, the starting point of the Kelvin scale, is the value recognized officially
by the National Institute of Standards and Technology (NIST).

The formulas are:

$$
\begin{aligned}
& \mathrm{C}=5 / 9\left(\mathrm{~F} \times 32^{\circ}\right)=\mathrm{K}-273.15^{\circ} \\
& \mathrm{F}=9 / 5 \mathrm{C}+32^{\circ}=9 / 5 \mathrm{~K}-459.67^{\circ} \\
& \mathrm{K}=5 / 9\left(\mathrm{~F}+459.67^{\circ}\right)=\mathrm{C}+273.15^{\circ}
\end{aligned}
$$

Table 30. Direction and Speed of True Wind - This table converts apparent wind to true wind. To use the table, divide the apparent wind in knots by the vessel's speed in knots. This gives the apparent wind speed in units of ship's speed. Enter the table with this value and the difference between the heading and the apparent wind direction. The values taken from the table are (1) the difference between the heading and the true wind direction, and (2) the speed of the true wind in units of ship's speed. The true wind is on the same side as the apparent wind, and from a point farther aft.

To convert wind speed in units of ship's speed to speed in knots, multiply by the vessel's speed in knots. The steadiness of the wind and the accuracy of its measurement are seldom sufficient to warrant interpolation in this table. If speed of the true wind and relative direction of the apparent wind are known, enter the column for direction of the apparent wind, and find the speed of the true wind in units of ship's speed. The number to the left is the relative direction of the true wind. The number on the same line in the side columns is the speed of the apparent wind in units of ship's speed. Two solutions are possible if speed of the true wind is less than ship's speed.

The table was computed by solving the triangle involved in a graphical solution, using the formulas:

$$
\begin{aligned}
& \tan \alpha=\frac{\sin \mathrm{B}_{\mathrm{A}}}{\mathrm{~S}_{\mathrm{A}}-\cos \mathrm{B}_{\mathrm{A}}} \\
& \mathrm{~B}_{\mathrm{T}}=\mathrm{B}_{\mathrm{A}}+\alpha \\
& \mathrm{S}_{\mathrm{T}}=\frac{\sin \mathrm{B}_{\mathrm{A}}}{\sin \alpha}
\end{aligned}
$$

in which $\alpha$ is an auxiliary angle, $\mathrm{B}_{\mathrm{A}}$ is the difference between the heading and the apparent wind direction, $\mathrm{S}_{\mathrm{A}}$ is the speed of the apparent wind in units of ship's speed, $\mathrm{B}_{\mathrm{T}}$ is the difference between the heading and the true wind direction, and $\mathrm{S}_{\mathrm{T}}$ is the speed of the true wind in units of ship's speed.

Table 31. Correction of Barometer Reading for Height Above Sea Level - If simultaneous barometer readings at different heights are to be of maximum value in weather analysis, they should be converted to the corre-
sponding readings at a standard height, usually sea level. To convert the observed barometer reading to this level, enter this table with the outside temperature and the height of the barometer above sea level. The height of a barometer is the height of its sensitive element; in the case of a mercurial barometer, this is the height of the free surface of mercury in the cistern. The correction taken from this table applies to the readings of any type barometer, and is always added to the observed readings, unless the barometer is below sea level.

The table was computed using the formula:

$$
\mathrm{C}=29.92126\left(1-\frac{1}{\operatorname{antilog}\left(\frac{0.0081350 \mathrm{H}}{\mathrm{~T}+0.00178308 \mathrm{H}}\right)}\right)
$$

in which C is the correction in inches of mercury, H is the height of the barometer above sea level in feet, and T is the mean temperature, in degrees Rankine (degrees Fahrenheit plus $459.67^{\circ}$ ), of the air between the barometer and sea level. At sea the outside air temperature is sufficiently accurate for this purpose.

Table 32. Correction of Barometer Reading for Gravity - The height of the column of a mercury barometer is affected by the force of gravity, which changes with latitude and is approximately equal along any parallel of latitude. The average gravitational force at latitude $45^{\circ} 32^{\prime} 40^{\prime \prime}$ is used as the standard for calibration. This table provides a correction to convert the observed reading at any other latitude to the corresponding value at latitude $45^{\circ} 32^{\prime} 40^{\prime \prime}$. Enter the table with the latitude, take out the correction, and apply in accordance with the sign given. This correction does not apply to aneroid barometers.

The correction was computed using the formula:

$$
\begin{aligned}
\mathrm{C}= & \mathrm{B}\left(-0.002637 \cos 2 \mathrm{~L}+0.000006 \cos ^{2} 2 \mathrm{~L}\right. \\
& -0.000050)
\end{aligned}
$$

in which C is the correction in inches, B is the observed reading of the barometer (corrected for temperature and instrumental errors) in inches of mercury, and L is the latitude. This table was computed for a standard height of 30 inches.

Table 33. Correction of Barometer Reading for Temperature - Because of the difference in expansion of the mercury column of a mercurial barometer and that of the brass scale by which the height is measured, a correction should be applied to the reading when the temperature differs from the standard used for calibration of the instrument. To find the correction, enter this table with the temperature in degrees Fahrenheit and the barometer reading. Apply the correction in accordance with the sign given. This correction does not apply to aneroid barometers.

The standard temperature used for calibration is $32^{\circ} \mathrm{F}$ for the mercury, and $62^{\circ} \mathrm{F}$ for the brass. The correction was
computed using the formula:

$$
C=-B \frac{m\left(T-32^{\circ}\right)-1\left(T-62^{\circ}\right)}{1+m\left(T-32^{\circ}\right)}
$$

in which C is the correction in inches, B is the observed reading of the barometer in inches of mercury, $m$ is the coefficient of cubical expansion of mercury $=0.0001010$ cubic inches per degree $F, 1$ is the coefficient of linear expansion of brass $=0.0000102$ inches per degree $F$, and $T$ is the temperature of the attached thermometer in degrees F . Substituting the values for $m$ and 1 and simplifying:

$$
\mathrm{C}=-\mathrm{B} \frac{\mathrm{~T}-28.630^{\circ}}{1.1123 \mathrm{~T}+10978^{\circ}}
$$

The minus sign before B indicates that the correction is negative if the temperature is more than $28.630^{\circ}$.

Table 34. Conversion Table for hecto-Pascals (Millibars), Inches of Mercury, and Millimeters of Mercury

- The reading of a barometer in inches or millimeters of mercury corresponding to a given reading in hecto-Pascals can be found directly from this table.

The formula for the pressure in hecto-Pascals is:

$$
P=\frac{B_{m} D_{g}}{1000}
$$

in which P is the atmospheric pressure in hecto-Pascals, $\mathrm{B}_{\mathrm{m}}$ is the height of the column of mercury in millimeters, $D$ is the density of mercury $=13.5951$ grams per cubic centimeter, and $g$ is the standard value of gravity $=980.665$ dynes. Substituting numerical values:

$$
\mathrm{P}=1.33322 \mathrm{~B}, \mathrm{and}
$$

$$
\mathrm{B}_{\mathrm{m}}=\frac{\mathrm{P}}{1.33322}=0.750064 \mathrm{P}
$$

Since one millimeter $=0.750064$ inches

$$
\mathrm{B}_{\mathrm{i}}=\frac{0.03937 \mathrm{P}}{1.33322}=0.0295300 \mathrm{P}
$$

in which $B_{i}$ is the height of the column of mercury in inches.

Table 35. Relative Humidity - To determine the relative humidity of the atmosphere, enter this table with the dry-bulb (air) temperature ( F ), and the difference between the dry-bulb and wet-bulb temperatures (F). The value taken from the table is the approximate percentage of relative humidity. If the dry-bulb and wet-bulb temperatures are the same, relative humidity is 100 percent.

The table was computed using the formula:

$$
\mathrm{R}=\frac{100_{\mathrm{e}}}{\mathrm{e}_{\mathrm{w}}}
$$

in which R is the approximate relative humidity in percent, e is the ambient vapor pressure, and $\mathrm{e}_{\mathrm{w}}$ is the saturation vapor pressure over water at dry-bulb temperature. Professor Ferrel's psychrometric formula was used for computation of e:

$$
\mathrm{e}^{\prime}-\left(0.000367 \mathrm{P}\left(\mathrm{t}-\mathrm{t}^{\prime}\right) \quad\left(1+\frac{\mathrm{t}-32^{\circ}}{1571}\right)\right)
$$

in which e is the ambient vapor pressure in millibars, $e^{\prime}$ is the saturation vapor pressure in millibars at wet-bulb tempera-
ture with respect to water, P is the atmospheric pressure (the millibar equivalent of 30 inches of mercury is used for this table), $t$ is the dry-bulb temperature in degrees Fahrenheit, and $\mathrm{t}^{\prime}$ is the wet-bulb temperature in degrees Fahrenheit.

The values of $\mathrm{e}_{\mathrm{w}}$ were taken from the International Meteorological Organization Publication Number 79, 1951, table 2, pages 82-83.

Table 36. Dew Point - To determine the dew point, enter this table with the dry-bulb (air) temperature (F), and the difference between the dry-bulb and wet-bulb temperatures ( F ). The value taken from the table is the dew point in degrees Fahrenheit. If the dry-bulb and wet-bulb temperatures are the same, the air is at or below the dew point.

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