CHAPTER 25

DOPPLER SONAR NAVIGATION

INTRODUCTION

2500. Doppler SONAR Velocity Logs

Significant improvements in maritime navigation can be obtained by the use of **Doppler Velocity Logs (DVLs)**. These acoustic sensors take advantage of the Doppler principle to provide a very accurate measure of the 3-dimensional velocity of a platform relative to the ground or to the ocean. The velocity over ground measured by the DVLs is much more accurate than can typically be achieved with a ship's **Inertial Navigation System (INS)**, which makes DVLs essential for missions requiring very accurate velocity and position.

Despite the ubiquity of Global Positioning Satellite (GPS) navigation today (see Chapter 22), the DVL still plays an important role in the suite of navigation instruments for surface vessels and a primary role for submerged vessels. The DVL provides an independent source of ship speed that is more reliable and has less random error than either the direct GPS velocity or the time derivative of GPS position. Also, the DVL is able to provide a navigation solution in situations where GPS cannot be used, for example when passing under bridges or near other structures, or in situations where the GPS signal is compromised due to electromagnetic jamming, spoofing, multipath, or unintentional interference from solar activity, geomagnetic storms, or other sources and for submerged vessels where GPS is not readily available. DVLs may also be used directly to aid an INS (see Chapter 20) by damping Schuler oscillations, aiding gyrocompassing, calibrating gyro and accelerometer bias and alignment errors, and controlling the medium-term growth of position error and the long-term growth of velocity error. Some DVLs can also provide measurements of current velocity near the vessel that may be useful in navigation and ship handling.

When anchoring large surface vessels without the aid of tugs, the speed over ground should be less than 0.3 knot to avoid accidental loss of the anchor and chain. The DVL is better than GPS at providing precise velocity information for this kind of operation. The DVL also can serve as a backup to laser- and radar-based docking aids for large surface vessels to maintain safe docking speeds in the event of failure of those systems.

The integration of GPS receivers into airborne and surface vehicles provides these vehicles with an increase in navigational accuracy that is unavailable to underwater vehicles. The navigation systems for underwater vehicles must rely on an inertial navigation system, a velocity log and gyrocompass, an array of transponders, or a combination of these systems. Although these navigation systems may be initialized with a GPS position fix, once the vehicle is submerged, the navigation system must operate autonomously. Recent improvements in the performance of bottom-tracking DVLs provide underwater vehicles with autonomous navigation accuracies on the order of 0.2 percent of distance traveled. However, vehicles equipped with a DVL are limited to operating at lower altitudes above the ocean bottom.

THE DOPPLER PRINCIPLE

2501. The Doppler Principle

This section introduces the Doppler principle and how it is used to measure relative radial velocity between different objects. The **Doppler effect** is a change in the observed sound pitch that results from relative motion. An example of the Doppler effect is the sound made by a train as it passes (Figure 2501). The train's whistle has a higher pitch as the train approaches an unmoving observer, and a lower pitch as it moves away. This change in pitch is directly proportional to how fast the train is moving. Therefore, by measuring the change in the pitch the speed of the train can be calculated.

Sound consists of pressure waves in air, water or solids. Sound waves are similar to shallow water ocean waves.

- **Waves**. Water wave crests and troughs are high and low water elevations. Sound wave crests and troughs are high and low air pressure.
- **Wavelength** (λ). The distance between successive wave crests.
- **Frequency** (f). The number of waves that pass in a unit of time.
- **Speed of Sound** (C). The speed at which the waves propagate.

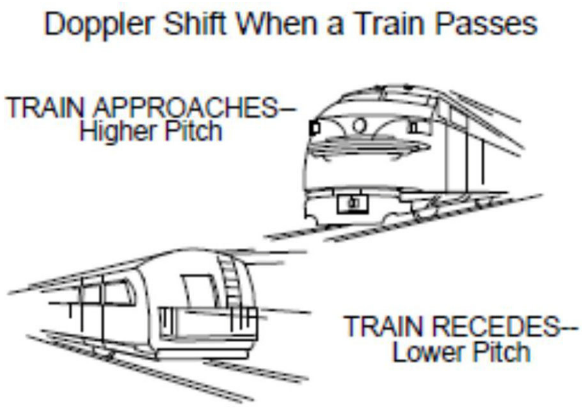

Figure 2501. When you listen to a train as it passes, you hear a change in pitch due to the Doppler shift. Courtesy of Johns Hopkins University - Applied Physics Laboratory.

Speed of Sound = Frequency x Wavelength

$$C = f\lambda$$

Example: A wave with a 300 kHz frequency and wavelength 5 mm will travel at (300,000 Hz) X (.005m) = 1500 meters per second.

Suppose while standing still near water, an observer sees eight waves pass in a given time interval. If the observer starts walking forward toward the waves more than eight waves will pass by in the same time interval, thus the wave frequency will appear to be higher. Similarly, if the observer walks in the opposite direction, away from the waves, fewer than eight waves pass by and the frequency appears lower. This is the Doppler effect. The **Doppler shift** is the difference between the frequency observed when standing still and that observed when moving.

Example. Standing still and you hear a frequency of 10 kHz, and then you start moving toward the sound source and hear a frequency of 10.1 kHz, then the Doppler shift is 0.1 kHz.

The equation for the Doppler shift in this situation is:

$$Fd = Fs(V/C)$$

Where:

- *Fd* is the Doppler shift frequency.
- *Fs* is the frequency of the sound when everything is still.
- *V* is the relative velocity between the sound source and the sound receiver (the speed at which you are walking toward the sound; m/s).
- C is the speed of sound (m/s).

Note that:

- If you walk faster, the Doppler shift increases.
- If you walk away from the sound, the Doppler shift is negative.
- If the frequency of the sound increases, the Doppler shift increases.

DVLs use the Doppler effect by *transmitting* sound at a fixed frequency and *listening* to echoes returning from either sound scatterers in the water or reflected off the ocean bottom. Sound scatters in all directions from scatterers or off the reflected surface. Much of the sound goes forward and is not reflected back. The small amount that reflects back is Doppler shift.

In the case of a DVL mounted on the underside of a vehicle, the vehicle is both a moving transmitter and receiver. The intermediate reflection of the acoustic signal off a water borne scatter or at the ocean bottom is treated as a stationary receiver immediately followed by a stationary transmitter. Because the DVL both transmits and receives sound, the Doppler shift is doubled.

$$Fd = 2Fs(V/C)$$

2502. DVL Transducers

The acoustic transmission and receiver in a DVL is accomplished with transducers. Acoustic transducers are transmitters that convert electricity to sound and also receivers that convert sound to electricity. The active elements in transducers are piezoelectric ceramic disks that expand or contract under the influence of an electric field. The electric field is applied through thin layers of silver deposited on the surfaces of the ceramic. When a voltage is applied, the disk gets thicker or thinner, depending on the polarity of the voltage. The ceramic disk is potted (encased) with polyurethane in a metal cup with a reflective backing material. Transducer quality is essential for data quality.

Figure 2502a. For more information on Janus.
http://en.wikipedia.org/wiki/Janus

DVL transducers are normally deployed using the Janus configuration, named for the Roman god Janus often depicted as having two faces looking in opposite directions (see Figure 2502a), which employs four ultrasonic beams, displaced 90° from each other in azimuth, with each directed obliquely at the ocean floor at angle θ_J from the vertical, to obtain true ground speed in the alongship and athwartship directions. The angle θ_J, typically 30°, is known as the Janus angle. Because the DVL combines information from all beams to determine velocity, there is no required orientation for the DVL relative to the ship. The two most common orientations are with opposing beams aligned to the alongship axis (Figure 2502b) or rotated 45° to have two beams pointing forward and two beams aft. Velocity components are measured in the direction of each beam using the Doppler effect on the backscattered acoustic signal. A development of the equation for these beam component measurements from the fundamental Doppler equation for sound propagation is presented in Section 2506.

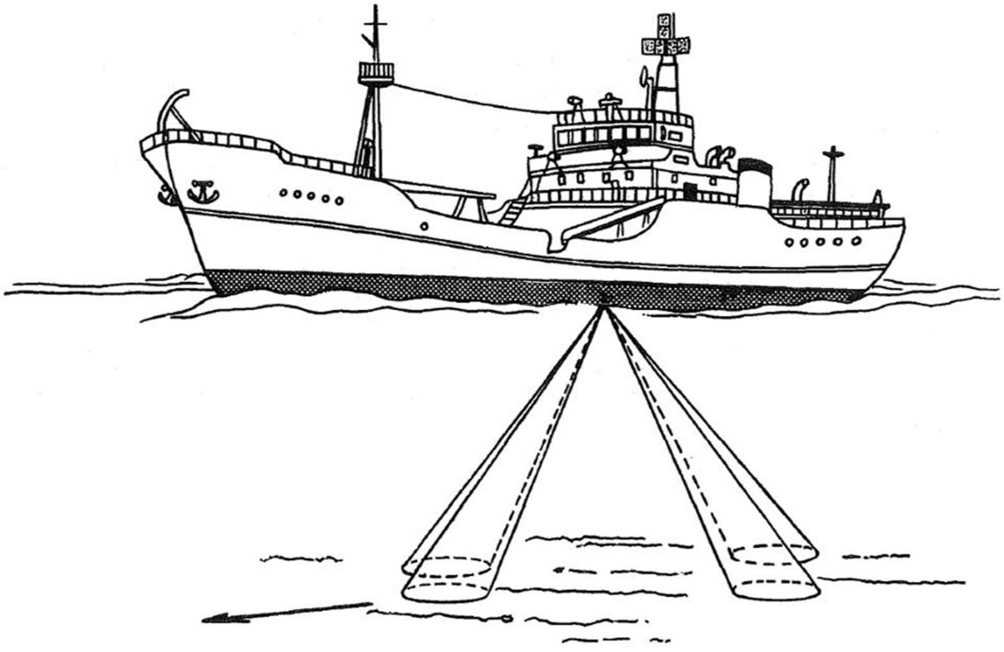

Figure 2502b. Janus configuration.

The advantage of the Janus configuration when the beams of one or both opposite pairs are pinged simultaneously is that the velocity differences within beam pairs that measure the horizontal velocity component are not affected by pitch or heave differences between beams that might occur were pinging to occur separately, nor by the large tilt- and heave-induced errors that could result if only one beam were used in each of the alongship and athwartship directions.

The Doppler theory equations for computing velocity for each DVL beam are given in Section 2506. Typical DVLs employ four-beams to measure three dimensional velocity of a vessel. A system such as this is over-determined as there are more beams than necessary. There are a number of potential solutions for going from a four beam velocity to a three dimensional velocity. The classic solution for solving over-determined systems is to use least squares (Brokloff, 1994).

An alternate approach is to use the redundant 4th beam to compute a direct measurement of a quality control output known as “error velocity” described in detail below. When the magnitude of the error velocity is unusually large for a particular ping and a bad measurement on a single beam can be identified as the cause, the remaining three beams can still be used to accurately determine the velocity vector.

The vector V_b of four beam-wise velocity components can be converted to the augmented velocity vector V_{inst} in the DVL instrument frame of reference by multiplying on the left by the beam-to-instrument transformation matrix B:

$$V_{inst} = B \cdot V_b$$

where $V_{inst} = \begin{bmatrix} u_{inst} \\ v_{inst} \\ w_{inst} \\ e \end{bmatrix}, \qquad V_b = \begin{bmatrix} V_{pd} \\ V_{sb} \\ V_{fb} \\ V_{ab} \end{bmatrix},$

and normally,

$$B = \begin{bmatrix} \frac{-1}{2\sin\theta_J} & \frac{1}{2\sin\theta_J} & 0 & 0 \\ 0 & 0 & \frac{1}{2\sin\theta_J} & \frac{-1}{2\sin\theta_J} \\ \frac{-1}{4\cos\theta_J} & \frac{-1}{4\cos\theta_J} & \frac{-1}{4\cos\theta_J} & \frac{-1}{4\cos\theta_J} \\ \frac{-1}{2\sqrt{2}\sin\theta_J} & \frac{-1}{2\sqrt{2}\sin\theta_J} & \frac{1}{2\sqrt{2}\sin\theta_J} & \frac{1}{2\sqrt{2}\sin\theta_J} \end{bmatrix}$$

and where u_{inst}, v_{inst}, w_{inst}, and e are respectively the nominally starboard, forward, upward, and error velocity components in the instrument frame and V_{pb}, V_{sb}, V_{fb} and V_{ab} are respectively the measured velocity components in the outward direction of the port, starboard, forward, and aft beams. (The beam labels correspond to their nominal azimuth directions in the first common orientation described above.)

Note. Many manufacturers use an inward-positive convention for beam velocities instead, which negates the signs of all elements of the beam-to-instrument matrix *B*. The outward-positive convention is used here to aid intuition when discussing the Doppler effect on the forward beam.

In general, the first three rows of *B* are the Moore-Penrose pseudoinverse of the 4×3 beam direction matrix having four rows, each row being the unit vector in the outward direction of a beam. These unit vectors can be determined during factory calibration. The elements of the B matrix shown above are the nominal values to be expected if there are no manufacturing deviations in beam orientation.

The last row of *B* is constructed to be orthogonal to the other three rows and normalized to make its magnitude (root-mean-square) match the mean of the magnitudes of the first two rows. It corresponds to the error velocity *e*, extra information in V_{inst} that augments the velocity vector comprising the first three of its elements. Its scaling was chosen so that the variance of the error velocity should have the same expected value as the portion of the variance of either of the two horizontal velocity components attributable to instrument noise rather than vessel motion. The expected value of the error velocity is zero.

The error velocity is useful for screening to exclude improbable measurements. In cases where the magnitude of the error velocity is unusually large compared to its standard deviation and the bad beam can be identified by, for example, the fact that its correlation coefficient is much lower than that of the other three beams, then the redundancy provided by having four beams allows the velocity vector to be determined by using the valid measurements from the three remaining beams. The algorithm that does this is equivalent to setting the error velocity e to zero and solving the equation corresponding to the last row of matrix *B* for a value to replace the rejected measurement from the bad beam, then continuing with processing as if all four beam measurements were known.

If the vessel's pitch and roll measurements are available from a gyrocompass, vertical gyro, or other sensor, they can be used to implement a coordinate frame rotation to give leveled horizontal velocity components in any sea state. This is known as the leveled ship frame. Likewise, heading from a magnetic compass, two-antenna GPS compass, gyrocompass, or INS can be used to calculate the velocity components in the geographic frame. These steps are shown in detail below.

If the DVL is not installed precisely aligned with the vessel axes, it is convenient to first rotate the measured velocity vector from the instrument to the ship frame using a constant alignment rotation matrix *A*:

$$V_{ship} = A \cdot V_{inst}$$

where $V_{ship} = \begin{bmatrix} u_{ship} \\ v_{ship} \\ w_{ship} \end{bmatrix}$, and

and nominally,

$$A = \begin{bmatrix} CH_A CR_A + SH_A SP_A SR_A & CH_A SP_A SR_A - SH_A CR_A & -CP_A SR_A & 0 \\ SH_A CP_A & CH_A CP_A & SP_A & 0 \\ CH_A SR_A - SH_A SP_A CR_A & -(CH_A SP_A CR_A + SH_A SR_A) & CP_A CR_A & 0 \end{bmatrix}$$

and the abbreviations *C* for **cos** and *S* for **sin** have been made in matrix ***A*** for notational convenience. The three angles H_A, P_A, and R_A respectively represent the heading, pitch, and roll *of the vessel* necessary to make the instrument be level and heading so the nominally-forward beam points north. For example, if the "forward" beam is installed to point 45° to port but is otherwise level, you would nominally use $H_A = 45°$, $P_A = 0°$, and $R_A = 0°$. Calibration of H_A, P_A, and R_A is discussed in the next section. When these angles are small, they essentially act as angle offsets that respectively subtract from the effective heading, pitch, and roll applied in the next velocity rotation steps discussed below. Of course, there are many possible ways to define the alignment matrix ***A***; this one has the advantage of keeping the adjustment angles well-defined and unambiguously communicated as ship attitudes. If the vessel's heading and attitude sensors are used with the DVL, then the rotation by ***A*** is necessary to align the velocity measurement with the axes of those sensors. Otherwise, this step is merely a convenience to allow the DVL's internal sensors to measure and report the vessel's heading and attitude instead of its own. In either case, the heading, pitch, and roll can be considered to be those of the vessel, not the instrument.

The next step is to rotate the velocity from the ship frame to the leveled ship frame by removing the effects of pitch P and roll R:

$$V_{level} = L \cdot V_{ship}$$

$$\text{where} \qquad V_{level} = \begin{bmatrix} u_{level} \\ v_{level} \\ w \end{bmatrix}, \text{ and}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos P & -\sin P \\ 0 & \sin P & \cos P \end{bmatrix} \cdot \begin{bmatrix} \cos R & 0 & \sin R \\ 0 & 1 & 0 \\ -\sin R & 0 & \cos R \end{bmatrix} = \begin{bmatrix} \cos R & 0 & \sin R \\ \sin P \sin R & \cos P & -\sin P \cos R \\ -\cos P \sin R & \sin P & \cos P \cos R \end{bmatrix}$$

According to the right-hand rule convention used here for the leveled ship frame, u_{level} is the drift to starboard, v_{level} is the forward speed of the vessel, and w is the upward velocity component.

If the velocity is desired in the geographic frame instead of the leveled ship frame, say for dead reckoning purposes, the velocity can be rotated yet again using the heading H:

$$V_{geo} = G \cdot V_{level}$$

$$\text{where} \qquad V_{geo} = \begin{bmatrix} u_E \\ v_N \\ w \end{bmatrix}, \text{ and}$$

$$G = \begin{bmatrix} \cos H & \sin H & 0 \\ -\sin H & \cos H & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and u_E and v_N are respectively the east and north components of the horizontal velocity.

2503. DVL Operational Errors

Long-term, systematic errors can be divided into instrument errors inherent to the technology, discussed in the next section, and operational errors that may be ameliorated by the user through proper installation, calibration, operation, and choice of accessory sensors, which we address here. Operational errors may be classified as *alignment errors, vessel motion induced errors, velocity of sound errors, interference errors, measurement outliers*, and *power loss errors*.

Alignment errors. If the DVL is not properly aligned with the compass's lubber's line, a dead-reckoned course will show a cross-track drift from the true course when compared to GPS. When using a magnetic compass, it may be difficult to distinguish this misalignment from compass variation. Most DVLs accept a calibration correction similar to H_A in the previous section to offset one or both of these angular errors.

When no pitch and roll measurements are available and the DVL is not level, or when the DVL is not properly aligned with the vertical fiducial of the pitch-roll sensor, the velocity measurement will be reduced by a factor equal to the cosine of the mean pitch angle (trim) or its residual error, assuming that the vessel is moving forward without leeway. This error can be detected by the DVL measuring a significant average vertical velocity, and the pitch error can be measured as the arctangent of the ratio of average vertical velocity (w) to average forward velocity v_{level}. This error is generally small; for example, an uncorrected trim (or list) of 4° will reduce the alongship (or athwartship) speed signal by about 0.25%. Calibration of the pitch alignment parameter P_A in the previous section can be accomplished by increasing the value of P_A by $\arctan(\langle w \rangle / \langle v_{level} \rangle)$. Similarly, R_A should be increased by $\arctan(\langle w \rangle / \langle u_{level} \rangle)$, but creating a significant average drift velocity $\langle u_{level} \rangle$ for the vessel may require use of a cross-current, side thrusters, or tugs in order to calibrate

Vessel motion induced errors. While alignment errors are caused by mean attitude offsets, there are additional analogous errors caused by dynamic fluctuations in pitch and roll when these are not measured and corrected. The alongship and athwartship velocity components averaged in the ship frame are biased low compared to averaging in the more stable leveled frame by a factor equal to the cosine of the standard deviation of the pitch and roll fluctuations, respectively. Even in the leveled ship frame or geographic frame, if an inadequate pitch/roll sensor is used that is not gyro-stabilized against contamination by wave-induced accelerations, there will be a similar bias effect multiplying the average horizontal velocity components by the cosine of the standard deviation of the resulting pitch and roll errors.

Analogous errors come from heading fluctuations. Dead reckoning calculation of progress along a course leg should be made in the more stable geographic frame to avoid the bias factor equal to the cosine of the standard deviation of the actual heading fluctuations. Even in the geographic frame, there is a cosine of the heading sensor errors factor that will bias the distance made good low.

Wave-induced motions of the vessel may cause speed fluctuations that may obscure the vessel's average speed over ground, but these fluctuations are not actual errors in the sense that they do reflect the ship's true motion. Some speed logs filter the output by providing a running average over several pings to reduce these fluctuations along with *random instrument errors*, with the unfortunate drawback of introducing a time lag to the speed measurement.

Velocity of sound errors. DVLs having transducers of the piston type directly measure the Mach number for the velocity component along each beam, which must be multiplied by the speed of sound to calculate the beam velocity. Therefore, the relative uncertainty in the sound speed results in a relative systematic error, or “scale factor bias,” of the same percentage. Although the sound speed varies over the water column and the deflection of sound rays by refraction depends primarily upon the mean sound speed over the water column, the sound speed needed by piston DVLs is that of the water at the transducer location. This is because of Snell's law of refraction, which implies that in horizontally-stratified water, the ratio of the sound speed to the sine of the sound ray inclination to the vertical is preserved throughout the water column. Hence, we must know the sound speed at the point where we know that the ray inclination is the Janus angle, which is at the transducer.

The speed of sound in bubble-free seawater depends upon temperature, salinity, and depth (of importance to submarines), but not frequency. The temperature dependence is the strongest but also the easiest to measure and compensate for in the DVL firmware using an empirical sound speed formula. Salinity is more inconvenient to measure but also less important, the sensitivity of sound speed being only about 0.1% per g/kg of salinity. However, in the brackish water of inlets and estuaries, the uncertainty in the salinity can be significant. In some installations an acoustic velocimeter may be used to directly measure the sound velocity in the vicinity of the transducer. Most commercially available DVLs accept a variety of different forms of sensor input to determine the sound speed factor used to convert Mach number to velocity.

A piston DVL can be mounted behind a flat window made of acoustically-transparent plastic with fresh water between the transducer and the window. The DVL is configured for a salinity of 0 g/kg and refraction at the window face automatically adjusts the Janus angle of the beams outside the window to compensate for the sound speed change due to the salinity of the sea water. Another scheme to avoid sound speed uncertainty is to use plastic prisms between the tilted transducers and the horizontal window, and employ a sound speed formula for the plastic as a function of temperature.

DVLs having phased array transducers not only have the advantage of smaller size by generating all four beams from a single aperture, but also of sound speed independence in the measurement of velocity components in the plane of the array. In fact, for horizontal motion of a level phased array in the direction of a beam azimuth, the Doppler shift is also independent of frequency, being (in hertz) half the ratio of the speed to the array stave spacing. The actual Janus angle doesn't matter to the measurement, although its sine does change slightly in proportion to sound speed and varies inversely with frequency over the bandwidth of the projected signal, always giving the same Doppler frequency shift for a given horizontal speed. In contrast, measurement of the velocity component perpendicular to the face of the phased array does depend upon sound speed, but since pitch is typically small, the propagation of speed-of-sound error into vessel speed is usually negligible.

Interference errors. Acoustic interference from other sonars having fundamental or harmonic frequencies near those used by the DVL can result in altitude error from false bottom lock, velocity bias, and other symptoms of jamming. This problem can be avoided by synchronizing multiple sonars to a ping schedule that avoids simultaneous pinging.

Outliers. While most DVL velocity errors have a nearly Gaussian distribution, unusual random events such as Rayleigh fades and ambiguity errors can occur that cause large errors with a higher probability than a bell-shaped curve would predict. For good performance, especially when dead reckoning, it is important to screen the measurements to remove outliers. DVLs may screen the velocity measurements automatically based upon loss of bottom lock, signal-to-noise ratio (SNR), relative intensity among beams, correlation coefficient, and/or error velocity, usually replacing bad measurements with the value from the previous good ping. Although threshold values for some of these screening tests may be under user control, the factory default settings are adequate under most conditions.

Power loss errors. The DVL requires a certain minimum SNR to detect the bottom and to distinguish the ping echo signal from thermal and ambient acoustic noise. The SNR, and thus the maximum altitude (maximum water depth for surface vessels), is affected by acoustic losses, some of which vary with environmental conditions. The most important of these are the temperature and salinity dependence of acoustic absorption by the seawater and variability in the bottom backscatter coefficient. Usually, increasing temperature decreases the maximum altitude for seawater, although at lower frequencies the best range performance is reached at moderate temperatures. At 30° C the maximum range may be reduced by as much as 30%. The maximum range is significantly greater in fresh water than in seawater.

At water depths near the maximum operational altitude of the DVL, near-surface bubble clouds may reduce the signal strength enough to cause dropped measurements, resulting in non-uniform sampling. If the motion of the vessel through the waves causes the timing of the presence of these bubbles and the resulting dropouts to be correlated with the wave-induced motion, the average of the remaining measurements may be biased. Three additional phenomena that can reduce signal strength in rough seas are (1) greater acoustic attenuation from the greater path length, (2) reduced bottom backscattering strength, both effects largest for a beam at its greatest angle from the vertical at maximum roll, and (3) rotation of the beam away from the direction toward which the sound was projected during the time interval between projection and receipt of the acoustic signal, which is worst at high altitudes and roll rates. Although the timing of these phenomena may vary relative to that of the bubbles, the dropouts they cause may also be correlated with the wave-induced motion of the vessel and thus sources of sampling bias. Therefore, the accuracy of the DVL output may be degraded when there are frequent dropouts in rough seas.

2504. Systematic Instrument Errors in DVLs

DVLs are subject to a number of long-term (i.e., systematic) errors besides those discussed in the previous section, the three most important kinds being *terrain, absorption*, and *sidelobe beam-coupling biases*.

Terrain bias. Because the bottom backscatter strength is a strong function of incidence angle, the side of the beam closer to the vertical is weighted more than the outer side, reducing the effective Janus angle and biasing the velocity low by some percentage. Although a typical value of this bias can be calibrated out, variability in the slope of the bottom backscatter strength function with incidence angle make the terrain bias depend upon bottom type. Flat, muddy bottoms generally give more terrain bias than rough rocky bottoms.

Absorption bias is similar to terrain bias in that it weights the inside of the beam more than the outside, but the cause is acoustic absorption over a shorter or longer path rather than differential bottom backscatter strength. Unlike terrain bias, which is independent of altitude, absorption bias is proportional to altitude and therefore worst at maximum depth. Absorption bias could be corrected if the water properties needed to estimate the acoustic absorption coefficient were known over the entire water column, but since they generally are not, most DVLs do not attempt to make a correction proportional to altitude.

Sidelobe beam-coupling bias is caused by acoustic leakage of the signal from opposite and neighboring beams through sidelobes of the beam pattern of the desired beam. This may become a problem when bottom slope, roll, or some other phenomenon increases the relative intensity of one or more of the unwanted beam contributions or reduces that of the beam being measured. For narrowband DVLs and for broadband DVLs at low speeds, this error behaves as a negative scale factor bias (i.e. the error is proportional to speed). For broadband DVLs at higher speeds, the velocity error is a periodic function of velocity, making its relative size less with increasing speed. Sidelobe beam-coupling bias can be avoided by pinging each beam separately in shallow water or by using different transmitted signals in different beams. Of the three kinds of systematic errors discussed only sidelobe beam coupling bias affects water-relative velocity measurements in the volumetric scattering mode.

All three kinds of systematic errors discussed above are reduced by using narrower beams, which requires either larger transducer diameter or higher frequency. DVLs can be calibrated against GPS to calculate a correction factor to remove these systematic errors, but in general, the correction will not universally apply. For submarines, DVL-aided INSs can calibrate scale factor bias against the more accurate INS accelerometer scale factor by doing turning or speed-changing maneuvers that create accelerations observable by a Kalman filter.

2505. Basic Design Considerations

Beamwidth. Beamwidth should be small enough to keep the systematic errors discussed in the previous section to reasonable levels, yet not so small that vessel motion causes signal loss in heavy seas. The width of the acoustic beam is inversely proportional to the diameter of the acoustic transducer. Most DVLs have transducer diameters between 15 and 30 wavelengths, the number generally lower at lower frequencies because of the greater cost of transducer size for larger systems and the longer acoustic travel time in deeper water causing greater SNR loss due to vessel motion.

Frequency. A high acoustic frequency is desirable for reducing transducer size and maximizing the Doppler phase shift. However, at higher frequencies the absorption loss is very nearly proportional to the square of the frequency. As the frequency is decreased, the loss tends to become linear. As the frequency is reduced to obtain greater maximum depth of operation in the bottom return mode, the transducer size grows increasingly large. Improvement in operating depth diminishes near 100 kHz while transducer size has increased considerably. Therefore, the frequency selected is a trade-off among desired maximum operating depth, transducer size, and cost. The region between 150 and 600 kHz is finding the greatest application.

Transmitted acoustic signal. The bandwidth of the transmitted signal is a programmable design variable. Narrowband transmissions achieve the greatest range at a particular frequency by minimizing thermal and ambient noise, at the expense of more erratic behavior due to stochastic signal fades that can occasionally prevent bottom lock. The narrowband mode also has additional sources of systematic error. In shallow to moderate water depths, where signal-to-noise ratio is not an issue, wider bandwidth produces more stable measurements with less random error. The signal bandwidth can be controlled independently of the duration of the projected pulse by phase-coding the signal. Most DVLs use a pulse duration that is a significant fraction of the travel time in order to fully ensonify the bottom, which greatly reduces the self-noise, one source of random error. In shallow water, the so-called “pulse-to-pulse coherent mode” having particularly low short-term error is also available, in which bottom echoes are received from each of two or more pulses before the next pulse in the transmission has been projected.

2506. Doppler Theory

A DVL relies on incoherent scattering of sound off the bottom, as opposed to specular reflection, so it tracks individual bottom scatterers that are *ensonified* by its beams. As a vessel moves over a sloping bottom, the velocity component measured by a beam will in general be unrelated to the time derivative of the range to the bottom along that beam, the latter being affected by new scatterers entering the beam at a slightly different range that have no velocity themselves.

The frequency of a plane wave signal of a particular wavelength is proportional to the rate of propagation of its phase fronts (pressure crests), which is the sound speed c in a reference frame moving with the water, but the rate is offset from c by the component of vessel velocity perpendicular to the phase fronts (i.e., parallel to the acoustic beam) in a reference frame moving with the vessel. For example, if the vessel is moving with vector velocity ***V*** and there is no current, then the forward beam-wise velocity component is $V_{fb} = \boldsymbol{V} \cdot \hat{\imath}_{fb} = u \sin \theta_J - w \cos \theta_J$ where $\hat{\imath}_{fb}$ is the unit outward vector in the direction of the forward-pointing beam. The frequency of the signal projected into the forward beam will be increased by the factor $f_w/f_p = c/(c - V_{fb}) = (1 - M_a)^{-1}$, where f_w is the frequency observed in the water frame, f_p is the frequency projected by the vessel, and M_a is the beam Mach number V_{fb}/c. When there is no current, *backscatter* does not change the frequency of the signal in the water frame, it simply reverses its direction. Upon returning to the vessel, the frequency will increase again by the factor $(1 + M_a)$, for a total of $\frac{1 + M_a}{1 - M_a}$. Subtracting 1 gives the Doppler shift factor:

$$\frac{\Delta f}{f_p} = \frac{2M_a}{1 - M_a} = \frac{2V_{fb}}{c - V_{fb}} = 2\frac{u \sin \theta_J - w \cos \theta_J}{c - u \sin \theta_J + w \cos \theta_J}$$

If this derivation is repeated with one or more water layers moving with the current, it will be found that the current only adds terms of order M_a^2 relative to the measurement, which are negligibly small.

For DVLs with piston transducers, the sound speed c at the transducer must be known to calculate the velocity from the Mach number. For phased arrays, the ratio $2(f_w/c) \sin \theta_J$ is equal to $1/(2d)$, where d is the stave spacing of the array and $f_w = f_p/(1 - M_a)$ is the frequency in the water frame, which is the average of the projected and received frequencies. The Doppler shift is therefore:

$$\Delta_f = 2f_w M_a = \frac{1}{2d}(u - w \cot \theta_j)$$

which is independent of frequency or sound speed for phased arrays in horizontal motion. Hence when $w = 0$, all frequency components of the signal experience the same Doppler *frequency shift* in phased arrays, whereas they experience the same relative *Doppler shift factor* in piston transducers.

The Doppler effect on a signal can alternatively be understood in the time domain as a small change in the time of arrival of repeated portions of the signal, no matter how narrow or wide its bandwidth. In a frame of reference fixed to the water, the point midway between the projection location and the point where the backscattered signal is received is known as the *phase center*. The displacement of the phase center over the lag t_L at which the projected signal repeats is $V\left(t_L - \frac{1}{2}\Delta t\right)$, where Δt is the amount of $V \cdot \hat{\imath}_{fb}\left(t_L - \frac{1}{2}\Delta t\right)$ to the sound displacement $c\,t_L$ in the same interval:

$$\frac{\Delta t}{t_L} = 2M_a\left(1 - \frac{1}{2}\frac{\Delta t}{t_L}\right) = \frac{2M_a}{1 + M_a} = \frac{2V_{fb}}{c + V_{fb}}$$

where the last two expressions come from solving the first equation for $\Lambda t/t_L$, and are equal to $\frac{\Delta f}{f_r}$, where $f_r = f_w(1 + M_a)$ is the received frequency. Solving the equation above for the forward beam velocity V_{fb}, we have:

$$V_{fb} = u \sin \theta_J - w \cos \theta_J = \left(\frac{c}{2t_L}\right)\frac{\Delta t}{1 - \frac{1}{2}\Delta t/t_L} = \left(\frac{c}{4f_0 t_L}\right)\frac{2f_0 \Delta t}{1 - \frac{1}{2}\Delta t/t_L}$$

with similar equations for the other beams. Most broadband DVLs having piston transducers measure $f_0 \Delta t$ using the phase of the demodulated signal autocorrelation function at or near the repeat lag t_L. The frequency f_0 is that of the local oscillator used for demodulation. The coefficient $\frac{c}{4f_0 t_L} = U_a$, known as the "ambiguity velocity," represents the velocity at which the phase is in radians, after correction for the non-linear denominator $1 - \frac{1}{2}\Delta t/t_L$. For narrowband DVLs, any lag can be used within the reciprocal of the signal bandwidth.

For phased array DVLs, it is useful to multiply Δt by the receive frequency $f_r = f_w(1 + M_a)$ to calculate the phase, which is the same at all frequencies:

$$f_r \Delta t = 2f_w t_L M_a = t_L \Delta f = \frac{1}{2U_{a0}}\left(u \sin \theta_0 - w \cos \theta_J\left(\frac{\sin \theta_0}{\sin \theta_J}\right)\right)$$

where $U_{a0} = \frac{c_0}{4f_0 t_L} = \frac{d}{t_L} \sin \theta_0$ is the ambiguity velocity at a standard sound speed defined by the choice of stave spacing d, and θ_0 is the nominal beam Janus angle, typically 30°. Broadband phased array DVLs use the phase measured from the demodulated signal autocorrelation function at or near t_L to measure $f_r \Delta t = t_L \Delta f$. The vertical and horizontal velocity components can be separated by respectively adding and subtracting the measurements from opposite beams (see Section 2502). Although the sensitivity of the vertical velocity component to sound speed is typically 33% greater for phased arrays than it is for pistons, there is no sound speed sensitivity at all for the nominally-horizontal components parallel to the array face.

2507. References

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