# CHAPTER 19 

# SIGHT REDUCTION 

## BASIC PROCEDURES

## 1900. Computer Sight Reduction

The purely mathematical process of sight reduction is an ideal candidate for computerization, and a number of different hand-held calculators, apps, and computer programs have been developed to relieve the tedium of working out sights by tabular or mathematical methods. The civilian navigator can choose from a wide variety of hand-held calculators and computer programs that require only the entry of the DR position, measured altitude of the body, and the time of observation. Even knowing the name of the body is unnecessary because the computer can identify it based on the entered data. Calculators, apps, and computers can provide more accurate solutions than tabular and mathematical methods because they can be based on precise analytical computations rather than rounded values inherent in tabular data.
U.S. Navy and Coast Guard navigators have access to a U.S. Government program called STELLA (System To Estimate Latitude and Longitude Astronomically; do not confuse with a similarly named commercial astronomy program). The Astronomical Applications Department of the U.S. Naval Observatory developed STELLA in response to a Navy requirement. The algorithms used in STELLA provide an accuracy of one arc-second on the Earth's surface, a distance of about 30 meters. While this accuracy is far better than can be obtained using a sextant, it does support possible naval needs for automated navigation systems based on celestial objects. These algorithms take into account the oblateness of the Earth, movement of the vessel during sight-taking, and other factors not fully addressed by traditional methods.

STELLA can perform almanac functions, position updating/DR estimations, celestial body rise/set/transit calculations, compass error calculations, sight planning, and sight reduction; on-line help and a user's guide are included. STELLA is now automatically distributed to each naval ship; other Navy users may obtain a copy by contacting:

## Superintendent

U.S. Naval Observatory

Code: AA/STELLA
3450 Massachusetts Ave. NW
Washington, DC, 20392-5420

## 1901. Tabular Sight Reduction

The process of deriving from celestial observations the information needed for establishing a line of position, LOP, is called sight reduction. The observation itself consists of measuring the altitude of the celestial body above the visible horizon and noting the time.

This chapter concentrates on sight reduction using the Nautical Almanac and Pub. No. 229: Sight Reduction Tables for Marine Navigation. Pub 229 is available on the NGA website. The method described here is one of many methods of reducing a sight. Use of the Nautical Almanac and Pub. 229 provide the most precise sight reduction practical, 0.'1 (or about 180 meters).

The Nautical Almanac contains a set of concise sight reduction tables and instruction on their use. It also contains methods and formulae for direct computation that may be used with a calculator or programmable computer.

The Air Almanac and NGA's Pub. 249, Sight Reduction Tables for Air Navigation, may also be used to reduce sights. Use of the Nautical Almanac's concise reduction tables, the Air Almanac, and Pub. 249 may all be used to reduce sights to a precision of 1'. The Nautical Almanac's concise reduction tables allow sight reduction by providing all celestial data in a single publication.

Reducing a celestial sight to obtain a line of position consists of six steps:

1. Correct the sextant altitude, $\boldsymbol{H s}$, to obtain observed altitude, Ho, (sometimes called true altitude).
2. Determine the body's Greenwich Hour Angle, GHA and declination, Dec.
3. Select an assumed position, $\boldsymbol{A P}$ and find its Local Hour Angle, LHA.
4. Compute altitude, $\boldsymbol{H c}$ and azimuth, $\mathbf{Z n}$, for the AP.
5. Compare the Hc and Ho.
6. Plot the line of position, $\boldsymbol{L O P}$.

The introduction to each volume of Pub. 229 contains information discussing:

1. The use of the publication for a variety of special celestial navigation techniques;
2. Interpolation.:
a. Explaining the second difference interpolation required in some sight reductions;
b. Providing tables to facilitate the interpolation process; and,
3. The publication's use in solving problems of great circle sailings.

Prior to using Pub. 229, carefully read this introductory material.

The goal of celestial navigation is to determine the navigator's position from the intersection of two or more $L O P \mathrm{~s}$ from observations of the altitudes of one or more celestial bodies and their geographic positions, GP, at the time of observation. The GP is the point on the Earth's surface that intersects the line from a celestial body to the Earth's center. In other words, at the $G P$, the celestial body is directly overhead. The locus of points on the Earth's surface where a body appears at a constant altitude forms a circle centered on the $G P$. So, when navigators observe the altitude of a body, they are determining the circle of constant altitude on which they are located. Except when a body's Ho approaches $90^{\circ}$, the portion of the circle of constant altitude plotted on a chart can be approximated by a line called the $L O P$. Assuming the $A P$ is within 30 ' of the true position, the maximum error accrued from estimating the circle of constant altitude with an LOP is $3 / 8$ mile for an Ho of $70^{\circ}$ and $3 / 4 \mathrm{mile}$ for an $H o$ of $80^{\circ}$.

In sight reduction, navigators choose an assumed position, $A P$, near, but usually not coincident with, their $D R$ position. The $A P$ 's latitude and longitude are chosen to correspond with the Local Hour Angle, LHA, and latitude which are entering arguments of the sight reduction tables in Pub. 229. From Pub. 229, the navigator determines the body's $H c$, its computed altitude, and its true azimuth, Zn , at the $A P$ at the time of observation. The difference between $H c$ and $H o$ is the altitude intercept, $\boldsymbol{a}$. The value of $a$, in minutes of arc, is equal to the distance, in nautical miles, between the circle of equal altitude on which the $A P$ is located and the one from which the body was observed.

The values of $a$ and $Z n$ are used to plot an $L O P$. First, the $A P$ is plotted on the chart or plotting sheet. A line is drawn through $A P$ at the angle $Z n$ with respect to true North. The $L O P$ is drawn perpendicular to this line. If Ho is greater than $H c$, then the $L O P$ is plotted in nautical miles from the $A P$ in the direction of $Z n$. If $H o$ is less than $H c$, then the $L O P$ is plotted in nautical miles from the $A P$ in the direction away from Zn .

## 1902. Selection of the Assumed Position (AP)

As mentioned above, the AP is chosen so the navigator can use the tabular values of a publication minimizing the need to interpolate. Thus, the AP is typically not your DR, and it is different for each object observed, even if the ship is stationary. The AP latitude is chosen to be the nearest whole degree in latitude to the DR latitude. The AP
longitude is that nearest the DR longitude resulting in a whole degree of LHA for the observed body. The tabular interval in the sight reduction tables of Pub. 229 for both latitude and LHA is one degree. Selecting the AP in this manner eliminates interpolation in LHA and latitude.

## 1903. Comparison of Computed and Observed Altitudes

The altitude intercept (sometimes just called intercept), $\boldsymbol{a}$, is the difference between the radii of the circles of equal altitude passing through the $A P$ and the observer's true position. The position with the greater altitude is on the circle of smaller radius and closer to the observed body's $G P$. In Figure 1904, the $A P$ is shown on the inner circle. Therefore, $H c$ is greater than $H o$. One minute of arc is equal to one nautical mile. Therefore, $a$ is expressed in nautical miles toward, $T$, or away, $A$, from the $G P$, as measured from the $A P$. If $H o$ is greater than $H c$, the $L O P$ intersects at right angles the line drawn from the $A P$ Towards the $G P$ at a distance of $a$ miles. If $H c$ is greater than $H o$, the line of position intersects at right angles the line drawn from the $A P$ Away from the $G P$ at a distance of $a$ miles. Useful mnemonics for remembering the relation between $H o, H c$, and $a$ are: HoMoTo for Ho More Towards, and C-G-A or Coast Guard Academy for Computed Greater Away.

## 1904. Plotting a Line of Position (LOP) and Fixing the Position

Plotting an LOP is done in four steps:

1. Plot the $A P$.
2. Draw a light dashed line at true azimuth $Z n$ through the $A P$.
3. Measure $a$ along this line.
4. Draw a line perpendicular to the azimuth line through this point.

This line is the $L O P$, which represents that segment of the circle of equal altitude passing through the navigator's true position. See Figure 1904. The navigator's true position is somewhere along this line, but with only one sight performed, we do not know where.

A celestial navigation fix is the place where two or more $L O P$ s intersect. When three or more $L O P \mathrm{~s}$ are used to make a fix, they will usually not all intersect at the same point due to limitations in the precision of the observations. Instead, they will form a 'cocked hat.' A fix may be obtained from plotting the $L O P$ s of multiple objects observed at approximately the same epoch (time) or from plotting the $L O P$ s of observations of one or more objects made at multiple epochs.

The position of the vessel will have changed significantly between the first and last observations for a fix from


Figure 1904. The basis for the line of position from a celestial observation.
observations made at multiple epochs. This type of fix is referred to as a running fix. To determine the ship's position at a particular epoch, $L O P$ s from earlier epochs must be advanced along the $D R$ track and $L O P$ s from later epochs must be retarded along the $D R$ track. To advance or retard an LOP:

1. Draw a light line segment from the $A P$ at the epoch of observation so that it intersects the $L O P$ at right angles.
2. Measure this line segment's $Z n$ and length.
3. At the $A P$ of the common epoch, draw a light line segment of the same length and at the same Zn .
4. At the non- $A P$ end of this second line segment, draw a line perpendicular to it.

This new line is the advanced or retired $L O P$. A running fix is not as accurate as a fix made from observations made at about the same epoch due to accumulated errors from sources such as steering errors and currents.

If a running fix is made from multiple observations of the same body, then enough time between the observations should be allowed to elapse so that the Zn of the body has precessed by at least $30^{\circ}$. A change in Zn is required to determine where two $L O P$ s cross. A difference of $90^{\circ}$ is ideal, but a variance of $30^{\circ}$ is adequate to provide a good fix.

## 1905. Sight Reduction Procedures

It is important to develop a practical procedure to
reduce celestial sights consistently and accurately. Sight reduction involves several steps. An accurate sight reduction requires that each step be concisely and accurately performed. Sight reduction tables reduce the mathematics involved as much as possible to addition and subtraction. Careless errors, however, can render the $L O P$ deduced from even the most skillfully measured sights inaccurate. The navigator must work methodically to avoid errors.

Naval navigators will most likely use OPNAV 3530/1, U.S. Navy Navigation Workbook, which contains "strip forms" to aid in the reduction of sights using either NGA Pub. 229 or Pub. 249 with either the Nautical Almanac or the Air Almanac. OPNAV 3530/1 also contains strip forms to aid in determining ship's latitude by Polaris and the local times of sunrise, sunset, moonrise, and moonset using data from either the Nautical Almanac or the Air Almanac. The Nautical Almanac includes a strip form designed specifically for use with its concise sight reduction tables. Use of other strip forms is authorized with the proviso that they become an official part of the record for the workbook being used.

Figure 1905 is a reproduction of the OPNAV 3530/1 strip form for sight reduction using the Nautical Almanac and Pub. 229. Working from top to bottom the entries are:

Date: The UT date of the sight.
Body: The name of the body whose altitude was measured. Indicate whether the upper or lower limb was measured if the body was the Sun or the Moon.

GMT (Greenwich Mean Time): The UT (GMT is an

| OPNAV 3530/40 (4-73) H.O. 229 NAUT ALM | NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 8-01) |
| :---: | :---: |
| Date | DATE/DR POSIT |
| Body |  |
| GMT |  |
| IC |  |
| D |  |
| Sum |  |
| hs |  |
| ha |  |
| Alt Corr |  |
| Add'l Corr Moon HP/Corr |  |
| Ho |  |
| GHA (h) |  |
| Incre ( $\mathrm{m} / \mathrm{s}$ ) |  |
| $\begin{aligned} & \text { v/v Corr } \\ & \text { SHA } \\ & \hline \end{aligned}$ |  |
| Total GHA |  |
| $\pm 360^{\circ}$ |  |
| $\mathrm{a} \lambda(+\mathrm{E},-\mathrm{W})$ |  |
| LHA |  |
| Tab Dec |  |
| d\#/d cor |  |
| True Dec |  |
| a LAT <br> Same Contrary |  |
| Dec Inc/d |  |
| Tens / DSD |  |
| Units / DSD Corr |  |
| Total Corr |  |
| Hc (Tab) |  |
| Hc (Comp) |  |
| Ho |  |
| a |  |
| Z |  |
| Zn |  |
| Fix Lat |  |
| Long | M |
| Fix Time |  |
|  | 1 $i$ <br> $i$ $i$ |
| Sounding |  |
| Signature |  |

Figure 1905. Sight reduction strip form for use with the Nautical Almanac and Pub 229.
outdated name for UT) of the observation. The UT is the Watch Time of the observations adjusted for the Watch Correction and Zone Description (see Chapter 20).

IC (Index Correction): The instrumental correction for the sextant used. Chapter 18 discusses determining its magnitude and sign.
$\boldsymbol{D}$ (Dip): Dip correction is a function of the height of eye of the observer and atmospheric refraction. Its magnitude is determined from the Dip Table on the inside front cover of the Nautical Almanac.

Sum: The sum of IC and D.
Hs (Sextant Altitude): The altitude of the body measured by the sextant.
$\boldsymbol{H a}$ (Apparent Altitude): The sum of $H s$ and the $I C$ and D corrections.

Alt. (Altitude) Correction: Every observation requires an altitude correction. This correction is a function of the apparent altitude of the body. The Nautical Almanac contains tables for determining these corrections. The tables for the Sun, planets, and stars are located on the inside front cover and facing page, pages A2 and A3. The tables for the Moon are located on the back inside cover and preceding page, pages xxxiv and xxxv.

These tables are based on observations taken under "standard" weather conditions; that is, temperatures near $50^{\circ} \mathrm{F}$ and air pressures near 1010 mb . If observations are taken in conditions that deviate much from this, an additional altitude correction is needed; see the Nautical Almanac table on page A4.

Note that the correction found on A4 is to be applied in addition to the corrections found on pages A2, A3, xxxiv or xxxv.

Add'l Corr/Moon HP Corr.: An additional correction is required for sights of Mars, Venus and the Moon. It adjusts for the phase and parallax of these bodies. The correction is a function of the body observed, the epoch of observation, and Ha. The corrections for Venus and Mars are listed inside front cover of the Nautical Almanac. These corrections change from year to year. The correction for the Moon is a function of the Moon's $H a$, its $H P$, and whether the upper, U , or lower, $L$, limb was observed. The tables for this correction are located inside the back cover and on the preceding page. Enter the table at the appropriate values for $H a$ and $H P$, and then choose the value associated with the $L$ or $U$ column as appropriate. If the upper limb was observed subtract 30', as well.

Ho (Observed Altitude): Add together Ha, the Altitude Correction, and the Additional Correction or Moon HP Correction, as appropriate. The result is the observed altitude, Ho .

GHA (Tabulated GHA): The tabulated value for the whole hour immediately preceding the time of the sight observation. For the Sun, the Moon, or a planet, extract the tabulated value for the Greenwich Hour Angle, GHA, of that body from the daily pages of the Nautical Almanac. For example, if the sight was obtained at $13^{\mathrm{h}} 50^{\mathrm{m}} 45^{\mathrm{s}}$ UT, extract the GHA value for $13^{\mathrm{h}}$. For a star sight reduction, extract the tabulated value of the GHA of

Aries $\Upsilon$.
Incre: The GHA increment is an interpolation factor, correcting for the time that the sight differed from the whole hour. For example, if the sight was obtained at $13^{\mathrm{h}} 50^{\mathrm{m}} 45^{\mathrm{s}}$ UT, this increment correction accounts for the 50 minutes and 45 seconds after $13^{\mathrm{h}}$. The increment value is tabulated in the Increments and Corrections tables on pages ii through xxxi in the Nautical Almanac. The entering arguments are the minutes and seconds after the hour. Select the correction from the appropriate column. Use the column labeled Aries for sights of stars.
$v / v$ Corr. / SHA: The true rate of motion in hour angle for the Moon and planets usually varies from the mean motion used to determine the increments. The parameter $v$ is the difference between the mean and true motion in arc minutes per hour. The change in hour angle arising from $v$ of the body at the time of the sight observation is accounted for with the $v$ correction. The value of $v$ for a planet sight is found at the bottom of the planet's column in the daily pages of the Nautical Almanac. The value of $v$ for the Moon is located directly beside the tabulated hourly GHA values in the daily pages of the Nautical Almanac. A body's $v$ is positive unless it is listed with a negative sign. Enter the value for $v$ on the left-hand side of the strip form. The $v$ correction is found using the Increments and Correction tables on pages ii through xxxi in the back of the Nautical Almanac. Enter the table using the minutes of the time of observation. Find the value in the " $v$ or $d$ ' columns corresponding to the value of $v$ for the time of observation. Enter the corresponding correction on the right-hand side of the strip form with the same sign as $v$.

The Sidereal Hour Angle ( $S H A$ ) is the difference between the GHA of a star and the GHA of Aries. The SHA of a star changes slowly. The SHA's of the 57 navigational stars are listed, in alphabetical order of the stars names, in the star column of the daily pages of the Nautical Almanac. The mean monthly SHA's of 173 stars, including the 57 navigational stars, are listed in order of SHA on pages 268 through 273 of the Nautical Almanac. Enter the SHA in place of the $v$ correction in the strip form if reducing a star sight.

Total GHA: The total GHA is the sum of the tabulated GHA, the GHA increment, and either the $v$ correction or the star's SHA.
$\pm \mathbf{3 6 0}{ }^{\circ}$ : Since the LHA will be determined from subtracting or adding the assumed longitude to the GHA, adjust the GHA by $360^{\circ}$ if needed to facilitate the addition or subtraction.

Rule of Thumb:

- In East Longitudes, LHA = GHA + Longitude $\left(-360^{\circ}\right.$ as necessary)
- In West Longitudes, LHA = GHA - Longitude $\left(+360^{\circ}\right.$ as necessary)

Example: For a longitude of $90^{\circ}$ East and GHA of $300^{\circ}$
LHA $=G H A+$ Longitude $-360^{\circ}\left(300^{\circ}+90^{\circ}=390^{\circ}-360^{\circ}=\right.$ $30^{\circ}$ ).
$\boldsymbol{a} \boldsymbol{\lambda}$ (Assumed Longitude): Choose an assumed longitude, a $\lambda$. If the vessel is west of the prime meridian, $\mathrm{LHA}=\mathrm{GHA}-a$ $\lambda$, where LHA is the Local Hour Angle. If the vessel is east of the prime meridian, $\mathrm{LHA}=\mathrm{GHA}+a \lambda$. The $a \lambda$ is chosen so that it is the longitude closest to that DR longitude where the LHA is a whole degree.

LHA (Local Hour Angle): The LHA is the hour angle of the observed body at a $\lambda$. The LHA is GHA - a $\lambda$, for west longitudes and GHA $+\mathrm{a} \lambda$ for east longitudes. Note that this should be a whole degree, else you have chosen the a $\lambda$ incorrectly.

Tab. Dec.: Obtain the tabulated declination for the Sun, Moon, stars, or planets from the daily pages of the Nautical Almanac. The declination values for the Sun, Moon, and planets are listed in hourly increments. Enter the declination value for the whole hour immediately preceding the sight for these bodies. For example, if the sight is at $12^{\mathrm{h}} 58^{\mathrm{m}} 40^{\mathrm{s}}$, enter the tabulated declination for $12^{\mathrm{h}}$. The declinations of the 57 navigational stars are listed, in alphabetical order of the stars names, in the star column of the daily pages of the Nautical Almanac. The mean monthly declinations of 173 stars, including the 57 navigational stars, are listed in order of SHA on pages 268 through 273 of the Nautical Almanac.
$\boldsymbol{d} / \boldsymbol{d}$ Corr.: The declinations of the Sun, Moon, and planets change with time. The parameter $d$ is the amount of change in declination in arc minutes per hour. The change in declination of the body at the time of the sight observation is accounted for with the $d$ correction. The value of $d$ for a planet sight is found at the bottom of the planet's column in the daily pages of the Nautical Almanac. The value of $d$ for the Moon is located directly beside the tabulated hourly declination values in the daily pages of the Nautical Almanac. Enter the value for $d$ on the left-hand side of the strip form. The sign of the $d$ correction is determined by the trend of declination value. For example, for a sight taken at $12^{\mathrm{h}} 30^{\mathrm{m}} 00^{\mathrm{s}}$. Compare the declination values for $12^{\mathrm{h}}$ and $13^{\mathrm{h}}$ and determine if the declination value has increased or decreased. If it has increased, the $d$ correction is positive, and if it has decreased, the $d$ correction is negative. The magnitude of the $d$ correction is found using the Increments and Correction tables on pages ii through xxxi in the Nautical Almanac. Enter the table using the minutes of the time of observation. Find the value in the " $v$ or $d$ " columns corresponding to the value of $d$ for the time of observation. Enter the corresponding correction on the right-hand side of the strip form. The rate of change in declination of the stars is so small that their sights do not require a $d$ correction.

True Dec.: The sum of the tabulated declination and the $d$ correction is the true declination.
aLAT (Assumed Latitude): Choose the whole degree of latitude closest to the vessel's DR latitude as the assumed latitude, $a L A T$. If the assumed latitude and declination are both north or both south, label the assumed latitude Same. If one is north and the other is south, label the assumed latitude Contrary.

DateDecl. inc. / $d$ : Two of the three arguments used to enter the main table of Pub. 229, LHA and $a L A T$, are whole degree values. The third argument, declination, is interpolated in Pub. 229. The method for interpolating declination is described on pages X-XIV of each volume of Pub. 229. The interpolation tables are located on the inside of the front cover and following page and inside of the back cover and preceding page of each volume of Pub. 229.

Interpolation is done using the declination increment, $d$. (This $d$ is not the same as the $d$ factor in the Nautical Almanac.) From the main table of Pub. 229 extract the value of $d$ for the tabular declination value preceding the body's declination. For example, if the body's declination is $30^{\circ}$ 35.1 , then record the tabular values in the row for Dec. $=$ $30^{\circ}$. If the value for $d$ is printed in italics and followed by a dot, then second-difference interpolation is required to maintain precision. In this case record the preceding and following entries for the value of $d$ as well. For example, for LHA $=28^{\circ}$, aLat $=15^{\circ}$ Same, Dec. $=16^{\circ}$, the entry for $d$ is $+0 . .^{\prime} 8$. The preceding entry is +2. . $^{\prime} 8 \bullet$, and the following entry is $-1.3^{\bullet}$. Record all three entries, in order.

Tens / DSD: If $d$ is greater than 10', then extract the interpolated value for the tens of $d$ from the interpolation tables in Pub. 229. Refer to the description for use of the interpolation tables on pages XI-XII of any volume of Pub. 229 for details.

Units / DSD Corr: Extract the interpolated value for the units of $d$ from the interpolation tables in Pub. 229. Refer to the description for use of the interpolation tables on pages XI-XII of any volume of Pub. 229 for details. If a second difference correction is required: subtract the value of the following entry for $d$ from the preceding value. For example, if the preceding entry is $+2 .{ }^{\prime} 8$, and the following entry is $-1 . ' 3$, then the result is $2 . ' 8-(-1 . ' 3)=4 . ' 1$. Use this value to enter in the appropriate part of the second difference portion of the interpolation table in Pub. 229. Refer to the description for the use of the second difference interpolation on page XIV of any volume of Pub. 229 for details. Add the second difference correction to the units correction before entering it in the strip form. Failure to include the second difference correction may result in an error of as much as 2.'1 in the final value of Hc.

Total Corr.: The sum of the tens and units corrections
is the total correction.
$\boldsymbol{H c}$ (Tab.): The tabulated value of $H c$ from the same entry in Pub. 229 from which $d$ was extracted.
$\boldsymbol{H c}$ (Comp.): The sum of $H c$ (Tab.) and the total corrections is Hc (Comp.).

Ho: The observed altitude calculated above.
$\boldsymbol{a}$ (Altitude Intercept): The absolute value of the difference between $H c$ and $H o$ is the altitude intercept, $a$. If $H o$ is greater than $H c$, then label $a$ as Toward. If $H c$ is greater than $H o$, then label $a$ as Away. Remember, "computed greater away."
$\boldsymbol{Z}$ (Azimuth Angle): The tabulated value of the azimuth angle, $Z$, is extracted from the same entry in Pub. 229 from which $H c$ and $d$ were extracted. Interpolation is not required.
$Z n$ (True Azimuth): The azimuth, $Z$, is the angular distance between the direction towards the observed body and the direction towards the elevated pole. The true azimuth, $Z_{n}$, is the angular distance measured eastward from the direction towards the North Pole to the direction towards the observed body. The value of $Z_{n}$ is a function of $Z$, LHA, and whether the observer is located north or south of the equator.

In northern latitudes:

$$
Z_{n}=\left\{\begin{array}{rr}
\mathrm{Z} & \mathrm{LHA}>180^{\circ} \\
360^{\circ}-\mathrm{Z} & \mathrm{LHA}<180^{\circ}
\end{array}\right.
$$

In southern latitudes:

$$
Z_{n}= \begin{cases}180^{\circ}-\mathrm{Z} & \mathrm{LHA}>180^{\circ} \\ 180^{\circ}+\mathrm{Z} & \mathrm{LHA}<180^{\circ}\end{cases}
$$

Fix., Lat., Long., Time: Enter the point of intersection and time when two or more $L O P$ s are plotted to determine a fix. The time of the fix is not necessarily the time of the sight because $L O P$ s may be advanced or retired. The diagram may be used to sketch the $\mathrm{Z}_{n}$ 's and $a$ 's used to determine the fix.

Sounding: If a sounding is available its value may be entered here.

Signature: The sight reduction is a part of the ship's official record and must be signed by the navigator.

## SIGHT REDUCTION

The section above discussed the basic theory of sight reduction and presented a method to be followed when reducing sights. This section puts that method into practice in reducing sights of a star, the Sun, the Moon, and planets.

## 1906. Reducing Star Sights to a Fix

In this section, we will reduce three star sights using the Nautical Almanac and Pub 229. The first of the three
sights will be reduced in step-by-step detail; the remaining two will not, since the technique is exactly the same. A strip form is utilized to aid in the reduction. A completed strip form showing the values from the three examples presented is shown in Figure 1906a.

On March 09, 2016, at the times indicated, the navigator takes and records the following sights (see the three column table with sextant altitude and zone time for Deneb, Antares and Alkaid):

| OPNAV $3530 / 40$ (4-73) H.O. 229 NAUT ALM | NAVIGATION WORKBOOK$\qquad$ |  |  | DATEDR Posit |
| :---: | :---: | :---: | :---: | :---: |
| Date | DATE/DR POSIT 9 Mar 2016 <br> $39^{\circ} \mathrm{N} 045^{\circ} 26.0^{\prime} \mathrm{W}$ | DATEDR Postr 9 Mar 2016 $39^{\circ} \mathrm{N} 045^{\circ} 26.0^{\prime} \mathrm{W}$ | DATEDR Posit 9 Mar 2016 $39^{\circ} \mathrm{N} 045^{\circ} 26.0^{\prime} \mathrm{W}$ |  |
| Body | Deneb | Antares | Alkaid |  |
| GMT | 08-58-27 | 09-02-14 | 09-06-32 |  |
| IC | +0.2' | +0.2' | +0.2' |  |
| D | -8.0' | -8.0' | -8.0' |  |
| Sum | -7.8' | -7.8' | -7.8' |  |
| hs | $50^{\circ} 34.4{ }^{\prime}$ | $23^{\circ} 57.2^{\prime}$ | $52^{\circ} 33.9{ }^{\prime}$ |  |
| ha | $50^{\circ} 26.6^{\prime}$ | $23^{\circ} 49.4{ }^{\prime}$ | $52^{\circ} 26.1^{\prime}$ |  |
| Alt Corr | -0.8' | -2.2' | -0.7' |  |
| Add'l Corr Moon HP/Corr | - | - | - |  |
| но | $50^{\circ} 25.8{ }^{\prime}$ | $23^{\circ} 47.2^{\prime}$ | $52^{\circ} 25.4{ }^{\prime}$ |  |
| GHA (h) | $287^{\circ} 26.6^{\prime}$ | $302^{\circ} 29.1^{\prime}$ | $302^{\circ} 29.1^{\prime}$ |  |
| Incre (m/s) | $14^{\circ} 39.2^{\prime}$ | $33.6{ }^{\prime}$ | $1^{\circ} 38.3^{\prime}$ |  |
| $\begin{aligned} & \text { v/v Corr } \\ & \text { SHA } \end{aligned}$ | $49^{\circ} 30.5^{\prime}$ | $112^{\circ} 23.9^{\prime}$ | $152^{\circ} 57.1^{\prime}$ |  |
| Total GHA | $351^{\circ} 36.3^{\prime}$ | $415^{\circ} 26.6^{\prime}$ | $457^{\circ} 4.5^{\prime}$ |  |
| $\pm 360^{\circ}$ | - | $055^{\circ} 26.6^{\prime}$ | $097^{\circ} 4.5^{\prime}$ |  |
| à ( $+\mathrm{E},-\mathrm{W}$ ) | $045{ }^{\circ} 36.3^{\prime} \mathrm{W}$ | $045^{\circ} 26.6^{\prime} \mathrm{W}$ | $045^{\circ} 4.5^{\prime} \mathrm{W}$ |  |
| Lha | $306{ }^{\circ}$ | $10^{\circ}$ | $52^{\circ}$ |  |
| Tab Dec | N 45 ${ }^{\circ} 20.2^{\prime}$ | S $26^{\circ} 27.8^{\prime}$ | N $49^{\circ} 13.8^{\prime}$ |  |
| d $/$ / d cor | - | - | - |  |
| True Dec | N 45 ${ }^{\circ} 20.2^{\prime}$ | S $26^{\circ} 27.8^{\prime}$ | N 49 ${ }^{\circ} 13.8^{\prime}$ |  |
| a LAT <br> Same Contrary | N $39^{\circ}$ same | N 39 ${ }^{\circ}$ cont | $\mathrm{N} 39^{\circ}$ same |  |
| Decinc/d | 20.2' / +10.8' | 27.8' / -59.4' | 13.8' / + 4.4' |  |
| Tens/DSD | - | - | - |  |
| Units / DSD Corr | - | - | - |  |
| Total Corr | +3.6' | -27.5' | +1.0' |  |
| Hc (Tab) | $50^{\circ} 10.5^{\prime}$ | $24^{\circ} 19.9{ }^{\prime}$ | $52^{\circ} 04.7{ }^{\prime}$ |  |
| Hc (Comp) | $50^{\circ} 14.1^{\prime}$ | $23^{\circ} 52.4{ }^{\prime}$ | $52^{\circ} 05.7^{\prime}$ |  |
| но | $50^{\circ} 25.8^{\prime}$ | $23^{\circ} 47.2^{\prime}$ | $52^{\circ} 25.4{ }^{\prime}$ |  |
| a | 11.7' toward | 5.2' away | 19.7 'toward |  |
| z | $63.3^{\circ}$ | $170.1^{\circ}$ | $57.3{ }^{\circ}$ |  |
| 2n | $63.3^{\circ}$ | $189.9^{\circ}$ | $302.7^{\circ}$ |  |
| Fix Lat |  |  |  |  |
| Long |  |  |  |  |
| Fix Time | " |  |  |  |
|  | - |  |  |  |
| Sounding | - , , i' | - . . , | ', , , | ! , , |
| Signature |  |  |  | usw, Mavicator |

Figure 1906a. Strip form for sight reduction of stars.

Height of eye is 68 feet and index correction (IC) is $+0.2^{\prime}$. The DR latitude for all sights is $39^{\circ} \mathrm{N}$. The DR longitude for all sights is $045^{\circ} 26.0^{\prime} \mathrm{W}$. See Figure 1906 b . Reduce the Deneb sight first.

Start by converting the sextant altitudes to the observed altitudes.

Determine the sum of the index correction and the dip correction. Go to the inside front cover of the Nautical Almanac to the table entitled "DIP." See Figure 1906c. This table lists dip corrections as a function of height of eye measured in either feet or meters. The dip table is a "critical table"; that is it is evaluated in intervals. To use it, find the interval of height of eye and read the corresponding correction. In the above problem, the observer's height of eye is 68 feet, which lies between the tabulated values of 67.1 to 68.8 feet; the corresponding correction for this interval is $-8.0^{\prime}$. Add the IC and the dip correction, being careful to carry the correct sign. The sum of the corrections here is $-7.8^{\prime}$. Apply this correction to the sextant altitude $\left(h_{s}\right)$ to obtain the apparent altitude $\left(h_{a}\right)$ of $50^{\circ} 26.6^{\prime}$.

Next, apply the altitude correction. Find the altitude correction table on the inside front cover of the Nautical Almanac next to the dip table. See Figure 1906c. This is also a critical table. The altitude correction varies as a function of both the type of body sighted (Sun, star, or planet) and the body's apparent altitude. For the problem above, enter the star altitude correction table; $h_{a}$ in this case was $50^{\circ} 26.6^{\prime}$. This value lies between the tabulated endpoints $48^{\circ} 45.0^{\prime}$ and $52^{\circ} 16.0^{\prime}$. The correction corresponding to this interval is $-0.8^{\prime}$. Applying this correction to $h_{a}$ yields an observed altitude of $50^{\circ} 25.8^{\prime}$.

Having calculated the observed altitude, determine the time and date of the sight in UT1 (Universal Time) or GMT (Greenwich Mean Time): Date $=09$ March 2016, DR Latitude $=39^{\circ} \mathrm{N}$, DR Longitude $=045^{\circ} 26.0^{\prime} \mathrm{W}$, Observation Time $=05-58-27$, Watch Error $=0$, Zone Time $=+3$, GMT $=08-58-27$, and GMT Date $=09$ March 2016.

Record the observation time and then apply any watch error to determine zone time. Then, use the DR longitude at the time of the sight to determine time zone description. In this case, the DR longitude indicates a zone description of +3 hours. Add the zone description to the zone time to obtain UT/GMT. It is important to carry the correct date when applying this correction (Note: this step, other than recording the DR, UT/GMT, and UT/GMT date, is not on the example strip form (Figure 1906a).

After calculating both the observed altitude and the UT/GMT time, calculate the star's Greenwich Hour Angle (GHA) and declination using the daily pages of the Nautical Almanac.

First, record the GHA of Aries from the March 09, 2016 daily page: $287^{\circ} 26.6^{\prime}$.

Next, determine the incremental addition for the minutes and seconds after 0800 from the Increments and Corrections table in the back of the Nautical Almanac. The increment for 58 minutes and 27 seconds is $14^{\circ} 39.2^{\prime}$.

Then, calculate the GHA of the star. Remember:

$$
\text { GHA }(\text { star })=\text { GHA } \Upsilon+\text { SHA (star })
$$

The Nautical Almanac lists the SHA of selected stars on each daily page. The SHA of Deneb on March 09, 2016: $49^{\circ}$ $30.5^{\prime}$. The Total GHA is $287^{\circ} 26.6^{\prime}+14^{\circ} 39.2^{\prime}+49^{\circ} 30.5^{\prime}=351^{\circ}$ $36.3^{\prime}$. If this were $360^{\circ}$ or larger, then subtract $360^{\circ}$.

Initially you must choose an assumed longitude, $a \lambda$, rather than using the ship's actual longitude. This is done because on of Pub 229's entering arguments is in whole degrees of LHA of the observed body. In order to get a whole degree of LHA, we will have to slightly change our DR longitude to a new, assumed longitude, $a \lambda$. The $a \lambda$ is chosen so that it is the longitude closest to that DR longitude where the LHA is a whole degree. Remember that if the vessel is west of the prime meridian, LHA = GHA $a \lambda$, and if the vessel is east of the prime meridian, LHA = GHA $+a \lambda$. In this example the vessel is in west longitude, so subtract its assumed longitude from the GHA of the body to obtain the LHA. The assumed longitude must end in 36.3', so that when subtracted from the Total GHA, a whole degree of LHA will result. Since the DR longitude was $045^{\circ}$ 26.0', the assumed longitude ending in $36.3^{\prime}$ closest to the DR longitude is $045^{\circ} 36.3^{\prime}$. Subtracting this assumed longitude from the calculated GHA of the star yields an LHA of $306^{\circ}$.

The next value of concern is the star's true declination. This value is found on the March 09 daily page next to the star's SHA (see Figure 1906b). Deneb's declination is N $45^{\circ} 20.2^{\prime}$. There is no $d$ correction for a star sight, so the star's true declination equals its tabulated declination.

The assumed latitude is determined from the whole degree of latitude closest to the DR latitude at the time of the sight. In this case, the assumed latitude is $\mathrm{N} 39^{\circ}$. It is marked "same" because both DR latitude and star's declination are north.

In order to find an object's altitude and azimuth using Pub 229, we need the ship's assumed latitude, the object's LHA, and its declination. We have this information for our example sight of Deneb. To recap, we have: the ship's assumed longitude ( $045^{\circ} 36.3^{\prime} \mathrm{W}$ ) and assumed latitude ( $39^{\circ} \mathrm{N}$ same); Deneb's LHA at a $\lambda\left(306^{\circ}\right)$; and Deneb's declination ( $\mathrm{N} 45^{\circ}$ 20.2'). We are now ready to use Pub 229 to find Deneb's altitude and azimuth.

Find the page in the Sight Reduction Table (using Pub 229 , for our example) corresponding to an LHA of $306^{\circ}$ and an assumed latitude of $\mathrm{N} 39^{\circ}$, with latitude same to declination. Enter this table with the body's whole degree of declination. In this case, the body's whole degree of declination is $45^{\circ}$. This declination corresponds to a tabulated
 of $45^{\circ}$; the true declination is $45^{\circ} 20.2^{\prime}$. Therefore, we need to interpolate to this increment (20.2') to obtain the comput-

2016 MARCH 7, 8, 9 (MON., TUES., WED.)


Figure 1906b. Left hand daily page of the Nautical Almanac for March 7, 8, \& 9, 2016.

## A2ALTITUDE CORRECTION TABLES $10^{\circ}-90^{\circ}-$ SUN,STARS,PLANETS

| OCT.-MAR. SUN APR.-SEPT. |  | STARS AND PLANETS |  | DIP |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| App. Lower Upper <br> Alt. Limb Limb | App. Lower Upper Alt. Limb Limb | $\underset{\text { Alt. }}{\text { App }} \text { Corr }^{n}$ | App. Additional Alt. Corr ${ }^{\text {n }}$ | $\underset{\text { Eye }}{\mathrm{Ht} \text {. of } \mathrm{Corr}^{\mathrm{n}}} \underset{\text { Eye }}{\mathrm{Ht} . \text { of }}$ | $\underset{\text { Eye }}{\text { Ht. of } \text { Corr }^{n}}$ |
| $\bigcirc 1$ | - 1 | $\bigcirc 1$ | 2016 | m ft. |  |
| $933+{ }^{\prime}{ }^{\prime} \cdot 8 \cdot{ }^{\prime}$ | $939+\stackrel{\prime}{10 \cdot 6-2 \mathrm{I}^{\prime} 2}$ | $955-\frac{1}{1}$ |  | $\begin{array}{lll}2.4 & \text {-2.8 } & 8.0\end{array}$ |  |
| $945+10.8-21.5$ | $950+10 \cdot 7-2 \mathrm{I} \cdot \mathrm{I}$ | 10 $07-53$ | VENUS | $2.6-2.80 .6$ | $1.5-2.2$ |
| $956+10 \cdot 9-21.4$ | 1002+10.7-21.1 | 10 $20-5.2$ | Jan. I-Dec. 3 | $2.8-3.0 \quad 9.2$ | $2 \cdot 0-2.5$ |
|  | $1014+10.9-20.9$ | Io $32-5.0$ |  | $3.0-3.109$ | $2.5-2.8$ |
| 10 $20+$ | $1027+11.0-20.8$ | Io $46-4.9$ | $60+\mathrm{O} \cdot \mathrm{I}$ | $\begin{array}{llll}3.2 & -3.2 & 10.5\end{array}$ | $3.0-3.0$ |
| 10 $33+11.3-21.0$ | 1040 + II I I - 20.7 | I0 $59-4.8$ | Dec. 4-Dec. 31 | $3.4-3.3$ I1.2 | See table |
| I0 $46+11.4-20.9$ | $1053+11.2-20 \cdot 6$ | II $14-4.7$ | Dec. $4^{- \text {Dee. }} 31$ | $3.6-3.4$ II.9 |  |
| II $00+$ II $5-20.8$ | II $07+$ II. $3-20.5$ | $\begin{array}{lll}\text { II } & 29 & -4.6\end{array}$ |  | $\begin{array}{lll}3.8 & -3.5 & 12.6\end{array}$ |  |
| II 15 + $11.6-20.7$ | II $22+$ II $44-20.4$ | II $44-4.5$ | $\mathrm{4I}^{1 \mathrm{I}} \stackrel{+0 \cdot 2}{+0 \cdot 1}$ | ${ }^{40}-3.6{ }^{13} 3$ | $20-7.9$ |
| II $30+11.7-20.6$ | II $37+$ II 5 - 20.3 | I2 $00-4.4$ |  | $\begin{array}{lll}4.3 & -3.7 & 14.1 \\ 4.5\end{array}$ | $22-8.3$ |
| II $45+\mathrm{II} \cdot 8-20 \cdot 5$ | II $53+11.6-20 \cdot 2$ | 12 17 | MARS | $45-3.8 \quad 149$ | $24-8.6$ |
| I2 OI $+11.9-20.4$ | $1210+11.7-20.1$ | I2 $35-4.2$ | Jan. I-Mar. II | $\begin{array}{lll}4.7 & -3.9 & 157\end{array}$ | $26-9.0$ |
| 12 $18+12.0-20.3$ | $1227+11.8-20.0$ | I2 $53-4 \cdot \mathrm{I}$ | Sept. 16-Dec. 3 I | $5.0-40 \quad 16.5$ | $28-9 \cdot 3$ |
| I2 $36+12 \cdot 1-20 \cdot 2$ | $1245+11.9-19.9$ | I3 $12-4.0$ |  | $5.2-4.158$ |  |
| $1254+\mathrm{I} 2 \cdot 2-20 \cdot \mathrm{I}$ | $1304+\mathrm{I} 2.0-19.8$ | $\begin{array}{llll}13 & 32 & -3.9\end{array}$ | 0 | $5.5-4.2{ }^{18.3}$ |  |
| $13 \mathrm{I} 4+\mathrm{I} 2 \cdot 3-20 \cdot 0$ | $1324+$ I2 I 10 I9 | $1353-3.8$ |  | $5^{\cdot 8}-4.3$ 19.1 |  |
| I3 $34+12$ | $1344+$ I2 | $14{ }^{16} \begin{aligned} & 16 \\ & -3.7\end{aligned}$ | Mar. I2-Apr. 30 | 6.1-4.3  | $32-10.0$ |
| I3 $55+12.4-19$ | $1406+12.2-19.6$ | I4 39 -37 | July 5-Sept. 15 | $6.3-4.4 \quad 21.0$ | $\begin{array}{ll}34 & -10 \cdot 3 \\ 36 & -10.6\end{array}$ |
| $14 \mathrm{I} 7+12.6-19$ | $1429+12.3-19.5$ $+12.4-19.4$ | $\begin{array}{llll}15 & 03 & -3.6\end{array}$ |  | $6.6 \begin{array}{lll}-4.5 \\ -4.6\end{array}$ | $36-10.6$ $38-10.8$ |
| 14 4I | $1453+12.5-19.3$ | 15 29-3 | ${ }^{0}+0.2$ | $6.9-4.622 .9$ | $38-10.8$ |
| 1505 | $1518+12.5-19.3$ | I5 $56-3.4$ | ${ }_{76} \mathrm{I}+\mathrm{O} \cdot \mathrm{I}$ | $7.2-478$ |  |
| 1531 | 1545 | I6 $25-33$ |  | $75-480$ | $40-\mathrm{II} \cdot \mathrm{I}$ |
| $1559+12.9-19.4$ | $16 \mathrm{I} 3+\mathrm{I} 2 \cdot 7-\mathrm{I9} \cdot \mathrm{I}$ | $1655-3.2$ | May I-July 4 | $7.9-4.926 .0$ | -II.4 |
| $1627+13.0-19.3$ | $1643+12 \cdot 8-19.0$ | $17{ }^{1} 27-3 \cdot 1$ |  | $8.2-5.027 .1$ | $44-\mathrm{II} \cdot 7$ |
| $1658+13 \cdot 1-19.2$ | $1714+12.9-18.9$ | I8 OI -3.0 | ${ }^{0} 4+0.3$ | $8.5-5.1-28.1$ | $46-11 \cdot 9$ |
| $1730+13.2-19.1$ | $1747+13.0-18.8$ | I8 37 <br> 18 | 60 +0.2 | $8.8-5.2$8.2 <br> 8.2 | $4^{8}-12 \cdot 2$ |
| $1805+13.3-19.0$ | $1823+13 \cdot 1-18.7$ | 19 16 ${ }^{-2 \cdot 8}$ | $80^{+0 \cdot \mathrm{I}}$ | -5.3 |  |
| $184 \mathrm{I}+13.4-18.9$ | $1900+13.2-18.6$ | $1956^{-2.7}$ |  | -5.4 | $2-1.4$ |
| 19 $20+13.5-18.8$ |  | $2040-2.6$ |  | $-5.5$ | $4-1 \cdot 9$ |
| $20.02+13.6-18.7$ | $2024+13.4-18.4$ | $21427-2.5$ |  | $\begin{array}{rrr}9.9 & -5.6 & 32 . \\ 10.3\end{array}$ | $6-2.4$ |
| $20.46+137-18.6$ | $21.4+135-18.3$ | 21 $27-2.4$ |  | $\begin{array}{llll}10.3 & -5.7 & 33.9 \\ 10.6\end{array}$ | $8-2 \cdot 7$ |
| $2134+13 \cdot 8-18 \cdot 5$ | $2159+13.6-18.2$ | 22 $17-2 \cdot 3$ |  | 10. -8 | $10-3 \cdot \mathrm{I}$ |
| $2225+13.9-18.4$ | $2252+137-18.1$ | $2409-2 \cdot 2$ |  | $1.4-59376$ | See table |
| 2225 2320 | $22.52+13.8-18.0$ | $\begin{array}{lll}24 & 09 \\ 25 & 12 & -2 \cdot 1\end{array}$ |  | $\begin{aligned} & 1.4 \\ & 1.8 \end{aligned}-6.0 \quad 37.0$ | $\leftarrow$ |
| $22^{20}+14 \mathrm{I}-18.2$ | $2451+13.9-17.9$ | $\begin{array}{lll}26 & 12 \\ 26 & 20\end{array}$ |  | $\begin{array}{ll}-6.1 & 40.1\end{array}$ |  |
| $2420+14.2-18 \cdot 1$ | 2451 | 26 20 <br> 27 34 |  | $12.2-6.2$ |  |
| 2524 2634 | $\begin{aligned} & 2558 \\ & 27 \text { II }\end{aligned}+14.1-17.7$ | $\begin{array}{lll}27 & 34-1.8 \\ 28 & 54\end{array}$ |  | $\begin{array}{lll}12.6 & -6.3 & 41.5 \\ 13.0 & 42.8\end{array}$ | $70-8.1$ $75-8.4$ |
| 2634 2750 | $\begin{aligned} & 2711 \\ & 28.31\end{aligned}+14.2-17.6$ | $\begin{array}{lll}28 & 54 \\ 30 & 22 & -17\end{array}$ |  | 13.0 -6.4 42.8 <br> 13.4   | $75-8.4$ $80-8.7$ |
| $2750+14.5-17.8$ 2913 | ${ }_{28}^{28} 51+14.3-17.5$ | $\begin{array}{lll}30 & 22 \\ 31 & 58\end{array}$ |  | 13.4 -6.5 | 80 85 |
| 2913 3044 | 29 5 $33+14.4-174$ | $\begin{array}{ll}31 & 58 \\ 33 & 43 \\ \text { - 1-5 }\end{array}$ |  | $\begin{array}{llll}13.8 & -6.6 & 45.5\end{array}$ | $85-8 \cdot 9$ |
| $3044+14.7-17.6$ | $3133+14.5-17.3$ | $3343-1.4$ |  | $\begin{array}{llll}14.2 & -6.7 & 46.9\end{array}$ | $90-9 \cdot 2$ |
| $3224+14.8-17.5$ | $3318+14.6-17.2$ | 35 38 <br> 8  |  | $14.7-6.8484$ | $95-9 \cdot 5$ |
| $3415+14.9-17.4$ | $3515+147-17 \cdot 1$ | $3745-\mathrm{I} \cdot 2$ |  | $15 \cdot 1-6.949 .8$ |  |
| $3617+15 \cdot 0-17.3$ | $3724+14.8-17.0$ | $40 \quad 06$ |  | $15.5-7 \cdot 0513$ | $100-97$ |
| $3834+150-17.3$ | $3948+14.9-16.9$ | $4^{42} 4^{-1}$ |  | $16.0-7.5$ | $105-9.9$ |
| $4 \mathrm{I} 06+15 \cdot 15 \cdot 1$ | $4228+149-16.9$ $+15.0-16.8$ | $45 \quad 34-1.0$ |  | $16.5-7.15$ | 110-10.2 |
| $4356+15.2-17.1$ $+15.3-17.0$ | $4529+150-16.8$ +15.15 .7 | 48 48 |  | $16.9-7.250 .8$ | II5 - 10.4 |
| $4707+15.4-16$ | $4852+15.2-16.6$ | $52{ }^{52} \begin{aligned} & 6\end{aligned}$ |  | $\begin{array}{lll}17.4 & -7.3 & 57.4\end{array}$ | $120-10.6$ |
| $5043+15.5-16.8$ | $5241+15.3-16.5$ | $\begin{array}{cc}56 & 09\end{array}$ |  | $179-74.4$ | $125-10 \cdot 8$ |
| $544^{+}+15.5-16.8$ | $5659+15.3-16.5$ | $60 \quad 26-0.6$ |  | $\begin{array}{lll}18.4 & -7.5 & 60.5\end{array}$ |  |
| $5921+15.6-16$ | $6150+15.4-16.4$ | $6506-0.5$ |  | $18.8-7.6$ 62.1 | 130-II•I |
| 6428+15 | $6715+15.5-16.3$ $+15.6-16.2$ | 70-09-0.4 |  | $19.3-7.703 .8$ | 135-11.3 |
| $7010+15$ | 7314 | $\begin{array}{llll}75 & 32-0.3\end{array}$ |  | $19.8 \begin{array}{lll} & -7.8 & 65.4\end{array}$ | $140-11.5$ |
| $7624+159$ | $7942+15$ | 8 I I2-0.2 |  |  | 145 - II.7 |
| $8305+16.0-16.3$ | $8631+15$ | $8703 \begin{array}{r}\text {-0.1 } \\ 0.0\end{array}$ |  | $\begin{array}{ccc}20.9 & -8.0 & 68.8\end{array}$ | $150-11.9$ |
| $9000{ }^{+}$ | $9000{ }^{+1}$ | 9000 |  | $21.4-8170.5$ | I55-12. I |

App. Alt. $=$ Apparent altitude $=$ Sextant altitude corrected for index error and dip.
ed altitude.
The difference between the tabulated altitudes for $45^{\circ}$ and $46^{\circ}$ is given in Pub. 229 as the value d ; in this case, $\mathrm{d}=$ $+10.8^{\prime}$. Note it is not in italics and followed by a dot, which makes the interpolation easier. There are two ways to do this interpolation. First, we could use the Interpolation Table on the inside front and back covers of Pub 229. We find the value of $+3.6^{\prime}$. Alternatively, we can easily compute the value. Express as a ratio the declination increment (in this case, 20.2') and the total interval between the tabulated declination values (in this case, 60') to obtain the 'distance' between the tabulated declination values represented by the declination increment. Next, multiply that by d, paying attention to the sign. In this case: $\left(20.2^{\prime} / 60^{\prime}\right) \times\left(+10.8^{\prime}\right)=+3.6^{\prime}$.

As expected, the two methods (using the Interpolation Table or computation) give the same result. Add 3.6' to the tabulated altitude to obtain the final computed altitude: $H_{c}$ $=50^{\circ} 14.1^{\prime}$.

It will be valuable here to review exactly what $h_{o}$ and $h_{c}$ represent. Recall the methodology of the altitude-intercept method. The navigator first measures and corrects an altitude for a celestial body. This corrected altitude, $h_{o}$, corresponds to a circle of equal altitude passing through the navigator's actual position whose center is the geographic position (GP) of the body. The navigator then determines an assumed position (AP) near, but not coincident with, his or her actual position; they then calculates an altitude for an observer at that assumed position (AP). The circle of equal altitude passing through this assumed position is concentric with the circle of equal altitude passing through the navigator's actual position. The difference between the body's altitude at the assumed position $\left(h_{c}\right)$ and the body's observed altitude ( $h_{o}$ ) is equal to the differences in radii length of the two corresponding circles of equal altitude.

In the above problem, therefore, the navigator knows that the equal altitude circle passing through his or her actual position is understood as: the difference between $h_{o}$ and $h_{c}$ is $50^{\circ} 25.8^{\prime}-50^{\circ} 14.1^{\prime}=11.7^{\prime}$ (which is 11.7 nautical miles) away from the equal altitude circle passing through his or her actual position. Since $h_{o}$ is greater than $h_{c}$, the navigator knows that the radius of the equal altitude circle passing through his or her actual position is less than the radius of the equal altitude circle passing through the assumed position. The only remaining question is what direction from the as-sumed position is the body's actual GP. Pub. 229 provides this final piece of information. This is the val-
ue for Z which is tabulated with the $h_{c}$ and $d$ values discussed above. In this case, enter Pub. 229 as before, with LHA, assumed latitude, and declination. Extract the value $Z=063.3^{\circ}$. Interpolation is not required. The relation between Z and Zn , the true azimuth, is as follows:

In northern latitudes:

$$
\begin{aligned}
& \text { LHA }>180^{\circ}, \text { then } Z_{n}=Z \\
& \text { LHA }<180^{\circ}, \text { then } Z_{n}=360^{\circ}-Z
\end{aligned}
$$

In southern latitudes:

$$
\begin{aligned}
& \text { LHA }>180^{\circ} \text {, then } Z_{n}=180^{\circ}-Z \\
& \text { LHA }<180^{\circ} \text {, then } Z_{n}=180^{\circ}+Z
\end{aligned}
$$

In this case, LHA $>180^{\circ}$ and the vessel is in northern latitude. Therefore, $\mathrm{Z}_{\mathrm{n}}=\mathrm{Z}=063.3^{\circ} \mathrm{T}$. The navigator now has enough information to plot a line of position.

Using the same technique, reduce the sights of Antares and Alkaid. The values for the sight reductions are shown in Figure 1906a.

## 1907. Reducing a Sun Sight

The example below points out the similarities between reducing a sun sight and reducing a star sight. It also demonstrates the additional corrections required for low altitude $\left(<10^{\circ}\right)$ sights and sights taken during non-standard temperature and pressure conditions.

On March 09, 2016, at 07-00-24 local time, at DR position L $39^{\circ} 11.0^{\prime} \mathrm{N} \lambda 45^{\circ} 22.0^{\prime} \mathrm{W}$, a navigator takes a sight of the Sun's lower limb. The navigator has a height of eye of 68 feet, the temperature is $88^{\circ} \mathrm{F}$, and the atmospheric pressure is 982 mb . The sextant altitude is $6^{\circ} 37.5^{\prime}$. There is an index correction of $+0.2^{\prime}$. Determine the observed altitude, computed altitude, and azimuth.

A completed strip form showing the values from this example is shown in Figure 1907a.

Apply the index and dip corrections to $h_{s}$ to obtain $h_{a}$. Because $h_{a}$ is less than $10^{\circ}$, use the special altitude correction table for sights between $0^{\circ}$ and $10^{\circ}$ located on the right inside front page of the Nautical Almanac.

Enter the table with the apparent altitude, the limb of the Sun used for the sight, and the period of the year. Interpolation for the apparent altitude is not required. In this case, the table yields a correction of +8.4 '. The correction's algebraic sign is found at the head of each group of entries and at every change of sign.

An additional correction is required because of the non-standard temperature and atmospheric pressure under which the sight was taken. The correction for these nonstandard conditions is found in the Additional Corrections table located on page A4 in the front of the Nautical Almanac.

First, enter the Additional Corrections table with the temperature and pressure to determine the correct zone letter: in this case, zone M. Then, locate the correction in the M column corresponding to the apparent altitude of $6^{\circ}$ 29.7'. Interpolate between the table arguments of $6^{\circ} 00.0^{\prime}$

| OPNAV 3530/40 (4-73) <br> HO. 229 NAUT ALM | NAVIGATION WORKBOOK OPNAV 3530/1 (Rev. 7-74) |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Date | DATE/DR POSIT 9 Mar 2016 $39^{\circ} 11^{\prime} \mathrm{N}, 045^{\circ} 22.0 \mathrm{~W}$ | DATE/DR POSIT 22 Mar 2016 DR not given in example | DATE/DR POSIT 9 Mar 2016 DR not given in example | DATEDR POSIT |
| Body | Sun LL | Moon LL | Mars |  |
| GMT | 10-00-24 | 21-01-04 | 08-58-34 |  |
| IC | +0.2 ${ }^{1}$ | $+0.2^{1}$ | +0.2 ${ }^{\prime}$ |  |
| D | -8.0' | -8.0' | -8.0' |  |
| Sum | -7.8 ${ }^{1}$ | -7.8 ${ }^{1}$ | $-7.8^{1}$ |  |
| hs | $6^{\circ} 37.5^{\prime}$ | $3^{\circ} 55.0^{\prime}$ | $29^{\circ} 43.0^{\prime}$ |  |
| ha | $6^{\circ} 29.7^{\prime}$ | $3^{\circ} 47.2^{\prime}$ | $29^{\circ} 35.2^{\prime}$ |  |
| Alt Corr | +8.4 ${ }^{\prime}$ | +56.1 ${ }^{\prime}$ | $-1.7^{\prime}$ |  |
| Add'l Corr Moon HP/Corr | +0.8 ${ }^{1}$ | +0.7 ${ }^{1}$ | +0.1 ${ }^{1}$ |  |
| Ho | $6^{\circ} 38.9^{\prime}$ | $4^{\circ} 44.0^{\prime}$ | $29^{\circ} 33.6^{\prime}$ |  |
| GHA (h) | $327^{\circ} 23.6^{\prime}$ | $319^{\circ} 28.2^{\prime}$ | $48^{\circ} 8.6^{\prime}$ |  |
| Incre ( $\mathrm{m} / \mathrm{s}$ ) | $6.0^{\prime}$ | +15.3 ${ }^{\prime}$ | $14^{\circ} 38.5^{\prime}$ |  |
| v/v Corr SHA | - | $15.0 /+0.4^{\prime}$ | $1.5^{\prime}$ |  |
| Total GHA | $327^{\circ} 29.6^{\prime}$ | $319^{\circ} 43.9^{\prime}$ | $62^{\circ} 48.6^{\prime}$ |  |
| $\pm 360^{\circ}$ | - | - | - |  |
| a $2(+E,-W)$ | $045^{\circ} 29.6^{\prime} \mathrm{W}$ |  |  |  |
| LHA | $282^{\circ}$ |  |  |  |
| Tab Dec | $54^{\circ} 14.9^{\prime}$ | N1 ${ }^{\circ} 59.8^{\prime}$ | $S 19^{\circ} 10.7^{\prime}$ |  |
| d\#/d cor | $-1.0 / 0.0^{1}$ | -9.4/-0.2 | $+0.2 /+0.2^{1}$ |  |
| True Dec | $54^{\circ} 14.9^{\prime}$ | N1 $1^{\circ} 59.6^{\prime}$ | $S 19^{\circ} 10.9^{\prime}$ |  |
| a LAT <br> Same Contrary | $39^{\circ}$ cont |  |  |  |
| Dec Inc/d | 14.9 / -38.6 |  |  |  |
| Tens / DSD | - |  |  |  |
| Units / DSD Corr | - |  |  |  |
| Total Corr | $-9.6{ }^{1}$ |  |  |  |
| Hc (Tab) | $6^{\circ} 44.1^{\prime}$ |  |  |  |
| Hc (Comp) | $6^{\circ} 34.5^{\prime}$ |  |  |  |
| Ho | $6^{\circ} 38.9^{\prime}$ |  |  |  |
| a | 4.4' toward |  |  |  |
| Z | $100.7^{\circ}$ |  |  |  |
| Zn | $100.7^{\circ}$ |  |  |  |
| Fix Lat |  |  |  |  |
| Long | M | M | M | M |
| Fix Time |  |  |  |  |
|  |  |  | $i \quad 1$ |  |
| Sounding |  |  | ' |  |
| Signature |  |  |  | USN, NAVIGATOR |

Figure 1907a. Strip for showing example values for Sun, Moon, and Mars.
and $7^{\circ} 00.0^{\prime}$ to determine the additional correction: $+0.8^{\prime}$. The total correction to the apparent altitude is the sum of the altitude and additional corrections: $+9.2^{\prime}$. This results in an $h_{o}$ of $6^{\circ} 38.9^{\prime}$.

| Date | 09 March 2016 |
| :--- | :--- |
| DR Latitude | N $39^{\circ} 11.01^{\prime}$ |
| DR Longitude | W 045 $22.0^{\prime}$ |
| Observation Time | $07-00-24$ |
| Watch Error | 0 |
| Zone Time | $07-00-24$ |
| Zone Description | +3 |
| UT/GMT | $10-00-24$ |

Next, determine the Sun's GHA and declination. Again, this process is similar to the star sights reduced above. See Figure 1907b. Notice, however, that SHA, a quantity unique to star sight reduction, is not used in Sun sight reduction.

Determining the Sun's GHA is less complicated than determining a star's GHA. The Nautical Almanac's daily pages list the Sun's GHA in hourly increments. In this case, the Sun's GHA at 1000 GMT on March 09, 2016 is $327^{\circ}$ 23.6'. The $v$ correction is not applicable for a Sun sight; therefore, applying the increment correction yields the Sun's GHA. In this case, the GHA is $327^{\circ} 29.6^{\prime}$.

Determining the Sun's LHA is similar to determining a star's LHA; see Section 1906 if more details are needed. In determining the Sun's declination, however, an additional correction not encountered in the star sight, the $d$ correction, must be considered. This is because the Sun moves in declination over the course of an hour, unlike that of the stars; the $d$ value is an interpolation factor for the Sun's declination. The bottom of the Sun column on the daily pages of the Nautical Almanac lists the $d$ value. The sign of the $d$ factor must be determined by noting from the Almanac whether the Sun's declination value is increasing or decreasing around the time of observation. If it is increasing, the factor is positive; if it is decreasing, the factor is negative. In the above problem, the Sun's declination value is decreasing throughout the day. Therefore, the $d$ factor is -1.0.

Having obtained the $d$ factor, enter the 00 minute increment and correction table. Under the column labeled " $v$ or $d$ corrn," find the value for $d$ in the left hand column. The corresponding number in the right hand column is the correction. Apply the correction to the tabulated declination. In this case, the correction corresponding to a $d$ value of -1.0 is $0.0^{\prime}$

The final step will be to determine $h_{c}$ and Zn . Enter $P u b$. 229 with an LHA of $282^{\circ}$, a declination of $S 4^{\circ} 14.9^{\prime}$, and an assumed latitude of $39^{\circ} \mathrm{N}$. The remaining values to determine $h_{c}$ and Zn are shown on the strip form (Figure 1907a).

## 1908. Reducing a Moon Sight

The Moon is easy to identify and is often visible during the day. However, the Moon's proximity to the Earth requires applying additional corrections to $h_{a}$ to obtain $h_{o}$. This section will cover Moon sight reduction.

At 21-01-04 UT, March 22, 2016, under standard meteorological conditions, the navigator obtains a sight of the Moon's lower limb. $h_{s}$ is $3^{\circ} 55.0^{\prime}$. Height of eye is 68 feet and the index correction is $+0.2^{\prime}$. Determine $h_{o}$, the Moon's GHA, and the Moon's declination.

A completed strip form showing the values from this example is shown in Figure 1907a. Also see Figure 1908a.

This example demonstrates the extra corrections required for obtaining $h_{o}$ for a Moon sight. Apply the index and dip corrections in the same manner as for star and Sun sights.

The altitude correction for the Moon comes from tables located on the inside back covers of the Nautical Almanac. Figure 1908b. In this case, the apparent altitude was $3^{\circ}$ 47.2'. Enter the altitude correction table for the Moon with the above apparent altitude. Interpolation is not required. The correction is $+56.1^{\prime}$. An arrow shows the correct location of this correction.

The altitude correction due to horizontal parallax (HP) is unique to Moon sights. The table for determining this correction is directly under the main altitude correction table for the Moon, on the back inside cover of the Nautical Almanac (see Figure 1908b). First, go to the daily page for March 22 at 21-00-00 UT/GMT (see Figure 1908a). In the column for the Moon, find the HP corresponding to 21-0000 . Its value is $54.3^{\prime}$ (no interpolation is required). Take this value to the Altitude Correction Tables for the Moon on the inside back cover of the Almanac. Notice that the HP correction columns line up vertically with the apparent altitude correction table columns. Find the HP correction column directly under the apparent altitude correction table heading corresponding to the apparent altitude. Enter that column with the HP from the daily pages. The column has two sets of figures listed under "L" and "U" for lower and upper limb respectively. In this case, trace down the "L" column until it intersects with the HP of 54.3'. The altitude correction due to the Moon's horizontal parallax is +0.7 ' in our example; an arrow shows the location of this correction. Interpolation is not required.

An additional correction based on non-standard weather condition is not applicable in this case because the sight was taken under standard temperature and pressure conditions.

The total correction to $h_{a}$ is the sum of all the corrections; in this case, this total correction is $+56.8^{\prime}$.

To obtain the Moon's GHA, enter the daily pages in the Moon column and extract the applicable data just as for a star or Sun sight. Determining the Moon's GHA requires an additional correction, the $v$ correction. The $v$ correction is needed because the Moon's motion is not close to uniform

2016 MARCH 7, 8, 9 (MON., TUES., WED.)


2016 MARCH 22, 23, 24 (TUES., WED., THURS.)


ALTITUDE CORRECTION TABLES $0^{\circ}-35^{\circ}-\mathrm{MOON}$

|  | $0^{\circ}$ | $5^{\circ}-9$ | $10^{\circ}-14^{\circ}$ | $15^{\circ}-19^{\circ}$ | $20^{\circ}-24^{\circ}$ | $25^{\circ}-29^{\circ}$ | $30^{\circ}-34^{\circ}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Corr ${ }^{\text {n }}$ | Corr | C | C0 | C | Co | Corr ${ }^{\text {n }}$ |  |
| 00 |  | $\mathbf{5}_{58}!2$ | $1060!$ | $1562: 8$ | 2062 | $2560 \cdot 8$ |  | 0 |
| 00 | $34 \cdot 5$ | $5{ }_{58,2}$ |  | ${ }^{15} 62.8$ |  | ${ }^{25} 60.8$ | 3058.9 | 0 |
| 10 | 36.5 | 58.5 | 22 | 62.8 | $62 \cdot 2$ | $60 \cdot 8$ | 58.8 | 10 |
| 20 | 38.3 | $58 \cdot 7$ | $62 \cdot 2$ | $62 \cdot 8$ | $62 \cdot 1$ | $60 \cdot 7$ | 8 8 | 20 |
| 30 | . | 58.9 | $62 \cdot 3$ | 62.8 | $62 \cdot \mathrm{I}$ | 60.7 | . 7 | 30 |
| 40 | 5 | I | $62 \cdot 3$ | $62 \cdot 8$ | $62 \cdot 0$ | $60 \cdot 6$ | . 6 | 40 |
| 50 | 42.9 | 59.3 | 62.4 | 62.7 | $62 \cdot 0$ | 60.6 | 58.5 | 50 |
| 00 | ${ }^{1}$ | ${ }^{6} 59.5$ | ${ }^{11} 62.4$ | 62.7 | ${ }^{21} 62.0$ | ${ }^{26} 60 \cdot 5$ | ${ }^{31} 58.5$ | 00 |
| 10 | 45.4 | 59.7 | 62.4 | $62 \cdot 7$ | 61.9 | 60.4 | 58.4 | 10 |
| 20 | $46 \cdot 5$ | 9 | 5 | 62.7 | 61.9 | 60.4 | 8.3 | 20 |
| 30 | 5 | - | 5 | 62.7 | I.9 | . 3 | 2 | 30 |
| 40 | 48.4 | 2 | 62.5 | $62 \cdot 7$ | 61.8 | $60 \cdot 3$ | 58.2 | 40 |
| 50 | $49 \cdot 3$ | 60.3 | 62.6 | $62 \cdot 7$ | 61.8 | $60 \cdot 2$ | $58 \cdot \mathrm{I}$ | 50 |
| 00 | ${ }^{2} 50$ | ${ }^{7} 60 \cdot 5$ | ${ }^{12} 62.6$ | ${ }^{1} 762.7$ | ${ }^{22} 61.7$ | ${ }^{27} 60 \cdot 1$ | $3^{2} 58.0$ | 00 |
| 10 | 8 | 6 | $2 \cdot 6$ | 62.6 | $6 \mathrm{I} \cdot 7$ | $60 \cdot 1$ | 579 | 10 |
| 20 | 5 | $60 \cdot 7$ | 6 | 62 |  | 0 | 7.8 | 20 |
| 30 | $52 \cdot 2$ | 60.9 | $62 \cdot 7$ | 62.6 |  | - 9 | 7.8 | 30 |
| 40 | 52.8 | $6 \mathrm{I} \cdot 0$ | 62.7 |  |  | 9 | 7 | 40 |
| 50 | 53.4 | 6I'I | $62 \cdot 7$ |  | $61 \cdot 5$ | $59 \cdot 8$ | 57.6 | 50 |
| 00 | $3_{53.9}$ | $8_{612}$ |  |  | ${ }^{23} 61 \cdot 5$ | ${ }^{28} \quad 59 \cdot 7$ | $3^{57.5}$ | 00 |
| 10 | 54.4 | $6 \mathrm{I} \cdot 3$ |  |  |  | 59.7 | . 4 | 0 |
| 20 | 54.9 |  |  |  | $6 \mathrm{I} \cdot 4$ | 59.6 | 57.4 | 20 |
| 30 | 3 |  | $62 \cdot 8$ | 6 | $6 \mathrm{I} \cdot 3$ | 9.5 | 57.3 | 30 |
| 40 | 55.7 |  | $62 \cdot 8$ | 62 | $61 \cdot 3$ | 59.5 | 57.2 | 40 |
| 50 | $56 \cdot \mathrm{I}$ | 61.6 | 62.8 | 62 | 61.2 | 59.4 | 57 I | 50 |
| 00 | ${ }^{4} 56.4$ |  | 14 | ${ }^{19} 62.4$ | ${ }^{24} 61 \cdot 2$ | ${ }^{29} 59.3$ | $4_{57.0}$ | 00 |
| 10 | 56.8 | $6 \mathrm{I} \cdot 8$ | 62.8 | 62 | - 1 | 59.3 | 56.9 | 10 |
| 20 | 5 | 6 I 9 | 62.8 | 62 | $6 \mathrm{I} \cdot 1$ | 59 | 56.9 | 20 |
| 30 | 57 | 61 | 62. | 6 | $6 \mathrm{I} \cdot 0$ | 59.1 | $56 \cdot 8$ | 30 |
| 40 | 5 | 62 | 62 | 6 |  | 59 |  | 40 |
| 50 | 58 | 62. | 62.8 | 62.2 | $60 \cdot 9$ | 59 | 56.6 | 50 |
| HP | L U | L U | L U | L U | L U | L U | L U | HP |
|  |  |  |  |  |  |  |  |  |
| 54.0 | 0.30 .9 | $0 \cdot 3$ | 0.4 | 0.5 | $0.6 \mathrm{I} \cdot 2$ | 0.71 .3 | 0.9 I. 5 | $4^{\circ}$ |
| 54 | 0.7 I. I | 0.7 I 2 | 0.8 | 0.8 I. 3 | 0.9 I. 4 | I-1 1.5 | $\begin{array}{lll}1.2 & \text { I }\end{array}$ | 543 |
| 54.6 |  |  | I•I I-4 | I. 2 |  | I.4 I.7 | I. 5 | 54.6 |
| 549 |  |  | I. 5 | I. 6 | I. 6 | I-81.9 | 1.92 | 54.9 |
| 55.2 | I. 8 |  |  | I.9 | 2.0 | 2.1 $2 \cdot \mathrm{I}$ | $2 \cdot 2$ | $55^{2}$ |
| 555 | 2.22 .0 | $2 \cdot 2$ |  |  | $2 \cdot 4$ | 2.42 .3 | $2 \cdot 5$ | 55.5 |
| 55.8 | 2.62 .2 | 2.62 .2 |  | $2 \cdot 7$ | 2.72 .4 | 2.82 .4 | 2.92 .5 | 55.8 |
|  | 3.02 .4 | 3.02 .5 |  | $3.02 \cdot 5$ | 3.12 .6 | 3.1 2.6 | 3.22 .7 | 56.1 |
| 56.4 | $3 \cdot 312 \cdot 7$ | $3 \cdot 427$ | $3 \cdot 4$ | 34 | 34 | $3 \cdot 5$ | 35 | 5 |
| 567 | 3.72 .9 | 3.72 .9 | $3 \cdot 8 \quad 2.9$ | 3.82 .9 | 3.83 .0 | $3 \cdot 8 \quad 3 \cdot 0$ | 3.9 | 56.7 |
| 57.0 | 4.1 3.1 | 4.1 $3 \cdot 1$ | 4.I 3 I | 4 I | 4.23 .2 | $4 \cdot 2$ | 4.2 | 57.0 |
| 573 | 4533 | 4533 | 4533 | 4533 | 4533 | 4534 | 4634 | 573 |
| 576 | 4935 | 4935 | 4935 | 4935 | 4935 | 4935 | 4936 | 57 |
| 579 | $5 \cdot 3 \cdot 3 \cdot 8$ | $5 \cdot 3 \quad 3 \cdot 8$ | $5 \cdot 23 \cdot 8$ | 5.237 | 5.237 | $5 \cdot 23 \cdot 7$ | $\begin{array}{llll}5 \cdot 2 & 3 & 7\end{array}$ | 57.9 |
| 58.2 | 5.640 | 5640 | 5640 | 5.640 | 563.9 | 5.63 .9 | 5.63 .9 | $58 \cdot 2$ |
|  | 6.0 | 6.0 | 6.0 | $6 \cdot$ | $6 \cdot$ | 594 I | 5.94 I | 58.5 |
| 58.8 | 6.444 | $6 \cdot 444$ | 6.444 | 6.344 | 6343 | 6.343 | 6.242 | 58.8 |
| 59.1 | 6.84 .6 | $6 \cdot 8 \quad 46$ | 6746 | 6.746 | 6745 | 6.645 | 6.644 | 59.1 |
| 59.4 | 7.248 | $7 \cdot 148$ | $7 \cdot 148$ | 7.1 4.8 | 7.047 | 7.047 | 6.946 | 59.4 |
| 597 | 7.5 | 75 | 7550 | 75 | 7449 | 7348 | 7.248 | 59.7 |
| 60.0 | 7953 | $79 \quad 53$ | 795.2 | 7.85 .2 | $7 \cdot 85 \cdot \mathrm{I}$ | 7750 | 7.649 | 60 |
| 60.3 | 8.355 | 8.355 | $8 \cdot 254$ | 8.254 | 8.1 5.3 | 8.052 | 7951 | 60.3 |
| 60.6 | 8.757 | 8.757 | 8.65 .7 | 8.65 .6 | 8.55 .5 | 8.45 .4 | 8.25 .3 | 60.6 |
| 60.9 | $9 \cdot 159$ | 9.059 | 9.0 59 | 8.95 .8 | 8.857 | 8.75 .6 | 8.654 | 60.9 |
| 61.2 | 9.56 .2 | 9.46 .1 | 9.46 .1 | 9.360 | 9.25 .9 | 9.1 $5 \cdot 8$ | 8.95 .6 | 61.2 |
| 615 | 9.86 .4 | 9.86 .3 | 9.76 .3 | 9.76 .2 | 956.1 | 9.45 .9 | 9.258 | 61.5 |

xxxiv

| DIP |  |
| :---: | :---: |
| $\left\lvert\, \begin{array}{\|cc} \text { Ht. of } \text { Corr }^{n} \\ \text { Eye } & \text { Ht. of } \\ \text { Eye } \end{array}\right.$ | $\underset{\text { Eye }}{\text { Ht. of }} \mathrm{Corr}^{\mathrm{n}}{ }_{\text {Eye }}^{\mathrm{Ht} \text { of }}$ |
| m ft. | m |
| $2.4-2.8 \quad 8.0$ | $9 \cdot 5-5^{1} 5^{31 \cdot 5}$ |
| 2.6 2.8 8.6 | $9.9-5.6{ }^{32 \cdot 7}$ |
| $2.8-2.9$ 9.2 | 10.3 -5.733.9 |
| $3 \cdot 0-3.19 .8$ | $10 \cdot 6-5783.1$ |
| $3.2-3.2$ I0.5 | II.O ${ }_{-5.9} \begin{aligned} & \text { S }\end{aligned}$ |
| $3 \cdot 4-3.3$ II. 2 | II. $4-5937.6$ |
| $3.6-3.3$ II.9 | II. $8-6.0{ }^{-6.9}$ |
| $3.8-3.4$ I2.6 | $2 \cdot 2-6.2{ }^{40 \cdot 1}$ |
| $-3.6{ }^{\text {I }} 3.3$ | ${ }^{12} 26-6.341 .5$ |
| 43314.1 | ${ }^{13} \cdot 0-6.4{ }^{42.8}$ |
| $45-3.814 .9$ | 13.4-6.5 44.2 |
| 47 -3.9 157 | $13.8-6.645 .5$ |
| $5.0-4.0$ I6.5 | $14.2-6.746 \cdot 9$ |
| $5 \cdot 2-4.174$ | $14.7-6.848 .4$ |
| $\cdot 5 \quad 4188$ | 15.1 4.49 .8 |
| $5.8-4.2$ I9.I | $15.5-6.951 .3$ |
| $6 \cdot \mathrm{I}-43$ 20.I | $16.0-7052.8$ |
| $6.3-4{ }^{-1.0}$ | I6.5 54.3 |
| $6.6-4.5 \quad 22.0$ | 16.9-7.2 ${ }^{1}$ |
| 6.9 22.9 | $17.4-7355$ |
| $7.2-4.783 .9$ | $17.9-7.458 .9$ |
| $7.5-4{ }^{249}$ | $18.4-7.560 .5$ |
| 26.0 | $18.8-7.662 .1$ |
| $8.2-5.027 .1$ | $19.3{ }^{-7.7} 63.8$ |
| $8.5-5.128 .1$ | $19.8-7.8 \quad 65.4$ |
| $8.8-5.229 .2$ | $20.4-7.967 .1$ |
| $9.2 \begin{array}{lll}-5 \cdot 3 & 30.4\end{array}$ | $20.9{ }^{-8.0} 68.8$ |
| $9.5-5.431 .5$ | $21.4{ }^{-8.1} 70.5$ |

## MOON CORRECTION TABLE

The correction is in two parts; the first correction is taken from the upper part of the table with argument apparent altitude, and the second from the lower part, with argument HP, in the same column as that from which the first correction was taken. Separate corrections are given in the lower part for lower (L) and upper(U) limbs. All corrections are to be added to apparent altitude, but $30^{\prime}$ is to be subtracted from the altitude of the upper limb.

For corrections for pressure and temperature see page A4.

For bubble sextant observations ignore dip, take the mean of upper and lower limb corrections and subtract $5^{\prime}$ from the altitude.

App. Alt. $=$ Apparent altitude $=$ Sextant altitude corrected for index error and dip.
throughout the year.
First, record the GHA of the Moon for 21-00-00 on March 22, 2016, from the daily pages of the Nautical Almanac. The increment correction is done as in the previous examples. In this case, it is $15.3^{\prime}$ because the sight was taken one minute and four seconds after the hour. From the daily page, record also the v correction factor, it is +15.0 . The $v$ correction factor for the Moon is always positive. To obtain the $v$ correction, go to the tables of increments and corrections. In the 1 minute table in the $v$ or d correction columns locate the correction that corresponds to $v=15.0$ '. The table yields a correction of $+0.4^{\prime}$. Adding this correction to the tabulated GHA and increment gives the final GHA as $319^{\circ}$ 43.9'.

Finding the Moon's declination is similar to finding the declination for the Sun or stars. The tabulated declination and the $d$ factor come from the Nautical Almanac's daily pages. Go to the daily pages for March 22, 2016; extract the Moon's declination and $d$ factor.

Record the declination and $d$ correction and go to the increment and correction pages to extract the proper correction for the given $d$ factor. In this case, go to the correction page for 1 minute. The correction corresponding to a $d$ factor of -9.4 is $-0.2^{\prime}$. It is important to extract the correction with the correct algebraic sign. The $d$ correction may be positive or negative depending on whether the Moon's declination value is increasing or decreasing in the interval covered by the $d$ factor. In this case, the Moon's declination value at 21-00-00 GMT on 22 March was $\mathrm{N} 01^{\circ}$ 59.8'; at 22-00-00 on the same date the Moon's declination was $\mathrm{N} 01^{\circ} 50.4^{\prime}$. Therefore, since the declination value was decreasing over this period, the $d$ correction is negative. Do not assume to determine the sign of this correction by noting the trend in the $d$ factor. For this problem, had the $d$ factor for 21-00-00 been a value more than 22, it would not indicate that the $d$ correction should be positive. Remember that the $d$ factor is analogous to an interpolation factor; it provides a correction to declination. Therefore, the trend in declination values, not the trend in $d$ values, controls the sign of the $d$ correction. Combine the tabulated declination and the $d$ correction factor to determine the true declination. In this case, the Moon's true declination is N $01^{\circ} 59.6^{\prime}$.

Having obtained the Moon's GHA and declination, calculate LHA and determine the assumed latitude. Enter the Sight Reduction Table with the LHA, assumed latitude, and calculated declination. Calculate the intercept and azimuth in the same manner used for star and Sun sights. (Note, for this example a DR was not provided, so these last steps are not shown on the strip form).

## 1909. Reducing a Planet Sight

There are four navigational planets: Venus, Mars, Jupiter, and Saturn. Reducing a planet sight is similar to reducing a Sun or star sight, but there are a few important
differences. This section will cover the procedure for determining $h_{0}$, the GHA and the declination for a planet sight.

On March 09, 2016, at 08-58-34 GMT, under standard meteorological conditions, the navigator takes a sight of Mars. H s is $29^{\circ} 43.0^{\prime}$. The height of eye is 68 feet, and the index correction is $+0.2^{\prime}$. Determine ho , GHA, and declination for Mars. A completed strip form showing the values from this example is shown in Figure 1907a.

The values on the filled-in strip form demonstrate the similarity between reducing planet sights and reducing sights of the Sun and stars. Calculate and apply the index and dip corrections exactly as for any other sight. Take the resulting apparent altitude and enter the altitude correction table for the stars and planets on the inside front cover of the Nautical Almanac.

In this case, the altitude correction for $29^{\circ} 35.2^{\prime}$ results in a correction of -1.7'. The additional correction for refraction is not applicable because the sight was taken at standard temperature and pressure; the horizontal parallax correction is not applicable to a planet sight, only for the Moon. All that remains is the correction specific to Mars or Venus. The altitude correction table on the inside front cover of the Nautical Almanac also contains this correction; once again see Figure 1906c. Its magnitude is a function of the body sighted (Mars or Venus), the time of year, and the body's apparent altitude. Entering this table with the data for this problem yields a correction of $+0.1^{\prime}$. Applying these corrections to h a results in an h o of $29^{\circ} 33.6^{\prime}$.

Determine the planet's GHA in the same manner as with the Sun. That is, extract the tabular GHA for the hour the sight was taken ( $48^{\circ} 08.6^{\prime}$ ). Then determine the incremental addition for the minutes and seconds after the hour from the Increments and Corrections table in the back of the Nautical Almanac ( $14^{\circ}$ 38.5 '). The only difference between determining the Sun's GHA and a planet's GHA lies in applying the $v$ correction. Recall that the $v$ correction was also needed for the Moon. As mentioned earlier, the $v$ correction is needed because the Moon and planets' motion are not close to uniform throughout the year.

Find the v factor at the bottom of the planets' GHA columns on the daily pages of the Nautical Almanac; see Figure 1906b. For Mars on March 09, 2016, the $v$ factor is 1.5. If no algebraic sign precedes the $v$ factor, add the resulting correction to the tabulated GHA. Subtract the resulting correction only when a negative sign precedes the $v$ factor. Entering the $v$ or $d$ correction table corresponding to 58 minutes yields a correction of 1.5 '. Remember, because no sign preceded the $v$ factor on the daily pages, add this correction to the tabulated GHA. The final GHA is $62^{\circ} 48.6^{\prime}$.

Read the tabulated declination directly from the daily pages of the Nautical Almanac; in our example it is S $19^{\circ}$ 10.7'. The $d$ correction factor is listed at the bottom of the planet column; in this case, the factor is 0.2 . Note the trend in the declination values for the planet; if they are increasing around the time the sight was taken, the correction factor is positive. If the planet's declination value is de-
creasing around this time, the correction factor is negative. Next, enter the $v$ or $d$ correction table corresponding to 58 minutes and extract the correction for a d factor of 0.2 . The correction in this case is $+0.2^{\prime}$. The true declination is $\mathrm{S} 19^{\circ}$ 10.9'.

From this point, reducing a planet sight is exactly the same as reducing a Sun sight. Having obtained the planet's

GHA and declination, the navigator can calculate LHA and determine the assumed latitude. Enter the Sight Reduction Table with the LHA, assumed latitude, and true declination. Calculate the intercept and azimuth in the same manner used for star and Sun sights. (Note, for this example a DR was not provided, so these last steps are not shown on the strip form).

## MERIDIAN PASSAGE

This section covers determining both latitude and longitude at the meridian passage of the Sun, or Local Apparent Noon (LAN). Latitude at LAN is a special case of the navigational triangle where the Sun is on the observer's meridian and thus the triangle becomes a straight north/south line. Should the navigator wish to plot the resulting LOP, it would be a straight east/west line running along the navigator's latitude. However, no complete sight reduction is necessary; the navigator need only to combine the Sun's zenith distance and its declination according to the rules discussed below. A new strip form, different from the one used in a sight reduction, is usually used (see Figure 1910).

Longitude at LAN is a function of the time elapsed since the Sun passed the Greenwich meridian. The navigator must determine the time of LAN and calculate the GHA of the Sun at that time. The following examples demonstrate determining latitude and longitude at LAN.

## 1910. Latitude at Meridian Passage of the Sun (Local Apparent Noon)

At 1208 ZT, March 09, 2016, a vessel's DR position is L $39^{\circ} 49.0^{\prime} \mathrm{N}$ and $\lambda 044^{\circ} 33.0^{\prime} \mathrm{W}$. The ship is on course $045^{\circ} \mathrm{T}$ at a speed of ten knots. (1) Calculate the first and second estimates of Local Apparent Noon. (2) The navigator actually observes LAN at 12-08-04 zone time. The sextant altitude at LAN is $45^{\circ} 54.0^{\prime}$. The index correction is $+0.2^{\prime}$ and the height of eye is 68 feet. Determine the vessel's latitude. A completed strip form showing the values from this example is shown in Figure 1910.

First, determine the time of meridian passage from the daily pages of the Nautical Almanac. It is found in the lower right corner of the right hand daily pages in Figure 1907b. In this case, the meridian passage for March 09, 2016, is 1210. That is, the Sun crosses the central meridian of the time zone at 1210 ZT. Next, determine the vessel's DR longitude for the time of meridian passage. In this case, the vessel's 1208 DR longitude is $044^{\circ} 33.0^{\prime}$ W. Determine the time zone in which this DR longitude falls and record the longitude of that time zone's central meridian. In this case, the central meridian is $45^{\circ} \mathrm{W}$. Enter the Conversion of Arc to Time table in the Nautical Almanac (page i, near the back) with the difference between the DR longitude and the central meridian longitude. The conversion for 27 ' of arc is $1^{\mathrm{m}} 48^{\mathrm{s}}$, which can be rounded to $2^{\mathrm{m}}$ of time. Sum these two
times. If the DR position is east of the central meridian (as it is in this case), subtract this time from the time of tabulated meridian passage. If the longitude difference is to the west of the central meridian, add this time to the tabulated meridian passage. In this case, the DR position is east of the central meridian. Therefore, subtract 2 minutes from 1210, the tabulated time of meridian passage. The estimated time of LAN is 12-08-00 ZT.

This first estimate for LAN took into account the vessel's movement. Therefore, it is unnecessary to conduct second estimate of LAN.

Solving for latitude requires that the navigator calculate two quantities: the Sun's declination and the Sun's zenith distance. First, calculate the Sun's true declination at LAN. The problem states that LAN is observed at 12-08-04. (Determining the exact time of LAN is covered in Section 1911.) Enter the time of observed LAN and add the correct zone description to determine GMT. Determine the Sun's declination in the same manner as in the sight reduction problem in Section 1906. In this case, the tabulated declination was $\mathrm{S} 4^{\circ} 10.0^{\prime}$, and $d$ is $-1.0^{\prime}$; the d correction (from the increments and corrections table) is $-0.11^{\prime}$. The true declination, therefore, is $\mathrm{S} 4^{\circ} 09.9^{\prime}$.

Next, calculate zenith distance. Recall from Chapter 17 Navigational Astronomy that zenith distance is simply $90^{\circ}$ observed altitude, $h_{o}$. Therefore, correct $h_{s}$ to obtain $h_{a}$; then correct $h_{a}$ to obtain $h_{o}$. This is done the same way as was done for reducing a Sun sight in Section 1907. Then, subtract $h_{o}$ from $90^{\circ}$ to determine the zenith distance. For our example, after applying the appropriate corrects, $h_{o}$ is $46^{\circ} 01.5^{\prime}$, and the zenith distance is $43^{\circ} 58.5^{\prime}$.

Your latitude is determined by applying the Sun's true declination to the zenith distance using the following rules:

- If DR lat and Dec (Sun) are same (e.g., both North or both South), then ADD
- If DR lat and Dec (Sun) are contrary (e.g., one North, one South), then SUBTRACT
- If DR lat and Dec (Sun) are same, and Dec is greater, then SUBTRACT

In this case, the DR latitude is N $39^{\circ} 49.0^{\prime}$ and the Sun's declination is $S 4^{\circ} 09.9^{\prime}$, which are contrary. Therefore, subtract the true declination from the zenith distance. The latitude for our example is $\mathrm{N} 39^{\circ} 48.6^{\prime}$.

## 1911. Longitude at Meridian Passage

Determining a vessel's longitude at LAN is straightforward. In the western hemisphere, the Sun's GHA at LAN equals the vessel's longitude. In the eastern hemisphere, subtract the Sun's GHA from $360^{\circ}$ to determine longitude. The difficult part lies in determining the precise moment of meridian passage.

Determining the time of meridian passage presents a problem because the Sun appears to hang for a finite time at its local maximum altitude. Therefore, noting the time of maximum sextant altitude is not sufficient for determining the precise time of LAN. Two methods are available to obtain LAN with a precision sufficient for determining longitude: (1) the graphical method and (2) the calculation method. The graphical method is discussed first below.

For about 30 minutes before the estimated time of LAN, measure and record several sextant altitudes and their corresponding times. See Figure 1911. Continue taking sights for about 30 minutes after the Sun has descended from the maximum recorded altitude. Increase the sighting frequency near the meridian passage. One sight every 20-30 seconds should yield good results near meridian passage; less frequent sights are required before and after.

Plot the resulting data on a graph of sextant altitude versus time and draw a fair curve through the plotted data. Next, draw a series of horizontal lines across the curve formed by the data points. These lines will intersect the faired curve at two different points. The $x$ coordinates of the points where these lines intersect the faired curve represent the two different times when the Sun's altitude was equal (one time when the Sun was ascending; the other time when the Sun was descending). Draw three such lines, and ensure the lines have sufficient vertical separation. For each line, average the two times where it intersects the faired curve. Finally, average the three resulting times to obtain a final value for the time of LAN. From the Nautical Almanac, determine the Sun's GHA at that time; this is your longitude in the western hemisphere. In the eastern hemisphere, subtract the Sun's GHA from $360^{\circ}$ to determine longitude. For a quicker but less exact time, simply drop a perpendicular from the apex of the curve and read the time along the time scale.

The second method of determining LAN is similar to the first. Estimate the time of LAN as discussed above, Measure and record the Sun's altitude as the Sun approaches its maximum altitude. As the Sun begins to descend, set the sextant to correspond to the altitude recorded just before the Sun's reaching its maximum altitude. Note the time when the Sun is again at that altitude. Average the two times. Repeat this procedure with two other altitudes recorded before LAN, each time presetting the sextant to those altitudes and recording the corresponding times that the Sun, now on its descent,


Figure 1911. Time of LAN.
passes through those altitudes. Average these corresponding times. Take a final average among the three averaged times; the result will be the time of
meridian passage. Determine the vessel's longitude by determining the Sun's GHA at the exact time of LAN.

## LATITUDE BY POLARIS

## 1912. Latitude by Polaris

Because Polaris is always within about $1^{\circ}$ of the North Pole, the altitude of Polaris, with a few minor corrections, equals the latitude of the observer. This relationship makes Polaris an extremely important navigational star in the northern hemisphere.

The corrections are necessary because over the course of 24 hours, Polaris appears to move in a small circle around the pole. When Polaris is at the exact same altitude as the pole, the correction is zero. When on the observer's meridian, that is at upper or lower culmination, the corrections are maximum. A special table, The Polaris Table, in the Nautical Almanac is used to determine the correction. The following example illustrates converting a Polaris sight to latitude. A new strip form, different from the one used in a sight reduction and LAN, is often used (see Figure 1912a).

At 23-18-56 GMT, on March 22, 2016, at DR Lat. $40^{\circ}$
$46.0^{\prime} \mathrm{N}, \lambda=043^{\circ} 22.0^{\prime} \mathrm{W}$, the observed altitude of Polaris $\left(h_{o}\right)$ is $40^{\circ} 52.1^{\prime}$. Find the vessel's latitude (see Figure 2310c).

To solve this problem, use the equation:

$$
\text { Latitude }=h_{o}-1^{\circ}+\mathrm{A}_{0}+\mathrm{A}_{1}+\mathrm{A}_{2}
$$

where $h_{o}$ is the sextant altitude $\left(h_{s}\right)$ corrected as in any other star sight; $1^{\circ}$ is a constant; and $\mathrm{A}_{0}, \mathrm{~A}_{1}$, and $\mathrm{A}_{2}$ are correction factors from the Polaris tables found on pages 274-276 of the Nautical Almanac (see Figure 1912b). These three correction factors are always positive. One needs the following information to enter the tables: LHA of Aries, DR latitude, and the month of the year. Therefore, LHA of Aries is determined similarly to the LHA of any other object. That is, first determine the GHA, then either add or subtract the DR longitude to compute LHA. Determining the GHA is done as with any other body. Extract the tabular

| OPNAV 3530/40 (4-73) LAT BY POLARIS | NAVIGATION WORKBOOK OPNAV 3530/1 (Rev 7-74) |
| :---: | :---: |
| Date | DATE/DR POSIT 22 Mar 2016 <br> $40^{\circ} 46^{\prime} \mathrm{N}, 043^{\circ} 22.0^{\prime} \mathrm{W}$ |
| Body | Polaris |
| GMT | 23-18-56 |
| IC |  |
| D |  |
| Sum |  |
| Hs |  |
| Ha |  |
| Alt Corr |  |
| Add'l Corr |  |
| Ho | $40^{\circ} 52.1^{\prime}$ |
| GHA $\gamma$ (h) | $165^{\circ} 52.3^{\prime}$ |
| Incre ( $\mathrm{m} / \mathrm{s}$ ) | $4^{\circ} 44.8^{\prime}$ |
| Total GHAY | $170^{\circ} 37.1^{\prime}$ |
| $\pm 360^{\circ}$ | - |
| DR $\lambda(+\mathrm{E}-\mathrm{W})$ | -43 $22.0^{\prime}$ |
| LHA $\gamma$ | $127^{\circ} 15.1^{\prime}$ |
| A0 | $54.9{ }^{\prime}$ |
| A1 | $0.5{ }^{\prime}$ |
| A2 | $0.9{ }^{\prime}$ |
| Sum | $56.3{ }^{1}$ |
| $1^{\circ} 00^{\prime}(-)$ | -1 ${ }^{\circ} 00.0^{\prime}$ |
| Total Corr | $-3.7{ }^{\prime}$ |
| Ho | $40^{\circ} 52.1^{\prime}$ |
| LAT | N $40^{\circ} 48.4^{\prime}$ |
| Time |  |
|  |  |
|  |  |
|  | $\begin{array}{l\|l} \hline i & i \\ \hline i & \\ \hline \end{array}$ |
| Sounding |  |
| Signature |  |

Figure 1912a. Sight reduction strip form for longitude by Polaris.

GHA for the hour the sight was taken $\left(165^{\circ} 52.3^{\prime}\right)$. Then determine the incremental addition for the minutes and seconds after the hour from the Increments and Corrections table in the back of the Nautical Almanac ( $4^{\circ} 44.8^{\prime}$ ). The total GHA of Aries is $170^{\circ} 37.1^{\prime}$. As described above, LHA is GHA $-\lambda$ for west longitudes and GHA $+\lambda$ for east longitudes. Because our example DR is in the western hemisphere, subtract the DR longitude from the GHA to obtain the LHA of Aries, $127^{\circ} 15.1^{\prime}$.

Next, enter the Polaris table with the calculated LHA of Aries at the time of observation ( $127^{\circ} 15.1^{\prime}$ ). The first correction, $\mathrm{A}_{0}$, is a function solely of the LHA of Aries. Enter the table column indicating the proper range of LHA of Aries; in this case, enter the $120^{\circ}-129^{\circ}$ column. The numbers on the left hand side of the $\mathrm{A}_{0}$ correction table represent the whole degrees of LHA $\Upsilon$; interpolate to determine the proper $\mathrm{A}_{0}$ correction. In this case, LHA $\Upsilon$ was $127^{\circ} 15.1^{\prime}$. The $A_{0}$ correction for $\mathrm{LHA}=127^{\circ} 54.7^{\prime}$ and the $\mathrm{A}_{0}$ correction for LHA $=128^{\circ}$ is $55.4^{\prime}$. The $\mathrm{A}_{0}$ correction for $127^{\circ} 15.1^{\prime}$ is 54.9'.

To calculate the $\mathrm{A}_{1}$ correction, enter the $\mathrm{A}_{1}$ correction table with the DR latitude, being careful to stay in the $120^{\circ}$ $129^{\circ}$ LHA column. There is no need to interpolate here; simply choose the latitude that is closest to the vessel's DR latitude. In this case, L is $40^{\circ} \mathrm{N}$. The $\mathrm{A}_{1}$ correction corresponding to an LHA range of $120^{\circ}-129^{\circ}$ and a latitude of $40^{\circ} \mathrm{N}$ is $+0.5^{\prime}$.

Finally, to calculate the $\mathrm{A}_{2}$ correction factor, stay in the $120^{\circ}-129^{\circ}$ LHA $\Upsilon$ column and enter the $\mathrm{A}_{2}$ correction table. Follow the column down to the month of the year; in this case, it is March. The correction for March is $+0.9^{\prime}$.

Sum the corrections, remembering that all three are always positive (56.3'). Subtract $1^{\circ}$ from the sum to determine the total correction ( $-3.7^{\prime}$ ), then apply the resulting value to the observed altitude of Polaris. The result is the vessel's latitude, $\mathrm{N} 40^{\circ} 48.4^{\prime}$.

## THE DAY'S WORK IN CELESTIAL NAVIGATION

## 1913. Celestial Navigation Daily Routine

The navigator need not follow the entire celestial routine if celestial navigation is not the primary navigation method. It is appropriate to use only the steps of the celestial day's work that are necessary to provide a meaningful
check on the primary fix source and maintain competency in celestial techniques.

The list of procedures below provides a complete daily celestial routine to follow. This sequence works equally well for all sight reduction methods, whether tabular, mathematical, computer program, or celestial navigation

POLARIS（POLE STAR）TABLES， 2016

| $\begin{aligned} & \text { LHA } \\ & \text { ARIES } \end{aligned}$ | $\begin{array}{r} 120^{\circ}- \\ 129^{\circ} \end{array}$ | $\begin{array}{r} 130^{\circ}- \\ 139^{\circ} \end{array}$ | $\begin{array}{r} 140^{\circ}- \\ 149^{\circ} \end{array}$ | $\begin{gathered} 150^{\circ}- \\ 159^{\circ} \end{gathered}$ | $\begin{array}{r} 160^{\circ}- \\ 169^{\circ} \end{array}$ | $\begin{array}{r} 170^{\circ}- \\ \quad 179^{\circ} \end{array}$ | $\begin{array}{r} 180^{\circ}- \\ 189^{\circ} \end{array}$ | $\begin{array}{r} 190^{\circ}- \\ 199^{\circ} \end{array}$ | $\begin{array}{r} 200^{\circ}- \\ 209^{\circ} \end{array}$ | $\begin{array}{r} 210^{\circ}- \\ 219^{\circ} \end{array}$ | $\begin{array}{r} 220^{\circ}- \\ 229^{\circ} \end{array}$ | $\begin{array}{r} 230^{\circ}- \\ 239^{\circ} \end{array}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $a_{0}$ | $a_{0}$ | $a$ 。 | $a_{0}$ | $a_{0}$ | $a_{0}$ | $a_{0}$ | $a$ 。 | $a_{0}$ | $a$ | $a_{0}$ | $a_{0}$ |
| － | － 1 | － 1 | － 1 | － 1 | － | － | － | － | － 1 | － | －$\quad 1$ | －＇ |
| 0 | － 49.9 | － 56.8 | $1 \quad 03.8$ | 1106 | 117.0 | I 22.9 | I 28.1 | I $32 \cdot 3$ | I 356 | I 37.8 | I 38.7 | I 38.5 |
| I | 506 | 57.5 | 045 | II 3 | 17.7 | 23.5 | 28.5 | $32 \cdot 7$ | 359 | 37.9 | $38 \cdot 8$ | $38 \cdot 4$ |
| 2 | $51 \cdot 3$ | $58 \cdot 2$ | $05 \cdot 2$ | Ir．9 | 18.3 | $24 \cdot 0$ | 29.0 | $33 \cdot 1$ | $36 \cdot 1$ | 38.0 | $38 \cdot 8$ | $38 \cdot 3$ |
| 3 | $52 \cdot 0$ | $58 \cdot 9$ | $05 \cdot 9$ | 12.6 | 18.9 | $24 \cdot 6$ | 29.5 | 33.4 | $36 \cdot 4$ | $38 \cdot 2$ | $38 \cdot 8$ | $38 \cdot 2$ |
| 4 | $52 \cdot 7$ | － 59.6 | 06.6 | 13.2 | 19.5 | $25^{\prime} 1$ | 29.9 | $33 \cdot 8$ | $36 \cdot 6$ | $38 \cdot 3$ | 38.8 | $38 \cdot 1$ |
| 5 | － 53.4 | I 00.3 | I 07.2 | $1 \quad 13.9$ | I 20.1 | I 25.6 | I 30.3 | I $34 \cdot 1$ | I 36.8 | I 38.4 | I 38.8 | I 38.0 |
| 6 | $54^{\circ}$ | Or ${ }^{\circ}$ | 07.9 | 14.5 | $20 \cdot 7$ | $26 \cdot 1$ | $30 \cdot 8$ | 34.4 | $37^{\circ}$ | $38 \cdot 5$ | $38 \cdot 8$ | $37 \cdot 8$ |
| 7 | 54.7 | $01 \cdot 7$ | 08.6 | 15.2 | $2 \mathrm{I} \cdot 2$ | $26 \cdot 6$ | 31.2 | 34.7 | 37.2 | 38.6 | $38 \cdot 7$ | 37.7 |
| 8 | $55 \cdot 4$ | 02.4 | 09.3 | 15.8 | $2 \mathrm{I} \cdot 8$ | 27.1 | $31 \cdot 6$ | $35^{\circ}$ | 37.4 | $38 \cdot 6$ | $38 \cdot 7$ | 37.5 |
| 9 | $56 \cdot \mathrm{I}$ | 03.1 | $09 \cdot 9$ | 16.4 | 22.4 | $27 \cdot 6$ | $32 \cdot 0$ | $35 \cdot 3$ | 37.6 | $38 \cdot 7$ | $38 \cdot 6$ | 37.3 |
| 10 | － 56.8 | 103.8 | 110.6 | I 17.0 | I 22.9 | $1 \quad 28.1$ | I $32 \cdot 3$ | I 35.6 | I 37.8 | I $38 \cdot 7$ | I 38.5 | I 37．1 |
| Lat． | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ | $a_{1}$ |
| 0 | 0.3 | 0.3 | $0 \cdot 3$ | $0 \cdot 4$ | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 | 0.6 | 0.6 | 0.6 |
| 10 | 4 | － 4 | 4 | $\cdot 4$ | $\cdot 4$ | $\cdot 5$ | $\cdot 5$ | $\cdot 5$ | － 6 | ． 6 | ． 6 | 6 |
| 20 | 4 | 4 | $\cdot 4$ | $\cdot 4$ | $\cdot 5$ | $\cdot 5$ | 5 | － 6 | － 6 | － 6 | － 6 | ． 6 |
| 30 | 5 | ． 5 | $\cdot 5$ | $\cdot 5$ | $\cdot 5$ | $\cdot 5$ | $\cdot 5$ | ． 6 | ． 6 | ． 6 | ． 6 | 6 |
| 40 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 45 | 6 | 6 | 6 | 6 | ． 6 | ． 6 | ． 6 | ． 6 | － 6 | ． 6 | 6 | 6 |
| 50 | 6 | 6 | 6 | 6 | ． 6 | 6 | － 6 | ． 6 | － 6 | ． 6 | 6 | 6 |
| 55 | 7 | $\cdot 7$ | 7 | 6 | － 6 | $\cdot 6$ | － 6 | ． 6 | － 6 | －6 | ． 6 | 6 |
| 60 | 7 | $\cdot 7$ | $\cdot 7$ | $\cdot 7$ | $\cdot 7$ | $\cdot 7$ | － 6 | ． 6 | －6 | ． 6 | ． 6 | 6 |
| 62 | 0.8 | 0.8 | 0.8 | 0.7 | 0.7 | 0.7 | 0.7 | 0.6 | 0.6 | 0.6 | 0.6 | 0.6 |
| 64 | ． 8 | ． 8 | ． 8 | ． 8 | $\cdot 7$ | $\cdot 7$ | $\cdot 7$ | － 6 | ． 6 | ． 6 | ． 6 | ． 6 |
| 66 | ． 8 | ． 8 | ． 8 | ． 8 | ． 8 | $\cdot 7$ | $\cdot 7$ | $\cdot 7$ | ． 6 | ． 6 | ． 6 | 6 |
| 68 | 0.9 | 0.9 | 0.9 | 0.9 | 0.8 | 0.8 | 0.7 | 0.7 | 0.6 | 0.6 | 0.6 | 0.6 |
| Month | $a_{2}$ $i$ | $a_{2}$ | ${ }_{\text {a }}$ | $a_{2}$ 1 | $a_{2}$ | $a_{2}$ | ${ }_{\text {a }}^{1}$ | ${ }_{\text {a }}^{1}$ | ${ }_{\text {a }}^{1}$ | ${ }_{\text {a }}^{1}$ | $a_{2}$ 1 | $a_{2}$ |
| Jan． | 0.6 | 0.6 | 0.6 | 0.6 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 |
| Feb． | 8 | 7 | 7 | 7 | ． 6 | ． 6 | $\cdot 6$ | $\cdot 5$ | $\cdot 5$ | － 5 | － 4 | 4 |
| Mar． | 0.9 | 0.9 | 0.9 | 0.8 | －8 | －8 | $\cdot 7$ | $\cdot 7$ | ． 6 | － 6 | ． 5 | 5 |
| Apr． | 1.0 | 1.0 | 1．0 | r．0 | 0.9 | 0.9 | 0.9 | 0.8 | 0.8 | 0.7 | 0.6 | 0.6 |
| May | 0.9 | 1.0 | r．0 | r．0 | 1．0 | 1．0 | 1．0 | 0.9 | 0.9 | 0.9 | ． 8 | 7 |
| June | 8 | 0.9 | 0.9 | 1.0 | $1 \cdot 0$ | 1.0 | 1.0 | 1.0 | 1.0 | 1.0 | 0.9 | 9 |
| July | 0.7 | 0.7 | 0.8 | 0.8 | 0.9 | 0.9 | 1.0 | 1.0 | $1 \cdot 0$ | 1.0 | 1.0 | 0.9 |
| Aug． | 5 | 6 | 6 | 7 | 7 | ． 8 | 0.8 | 0.9 | 0.9 | 0.9 | 1.0 | 1.0 |
| Sept． | 4 | 4 | 4 | 5 | ． 6 | ． 6 | 7 | $\cdot 7$ | ． 8 | ． 8 | 0.9 | 0.9 |
| Oct． | 0.3 | 0.3 | 0.3 | 0.3 | 0.4 | 0.4 | 0.5 | 0.5 | 0.6 | 0.7 | 0.7 | 0.8 |
| Nov． | 2 | 2 | 2 | － 2 | 3 | 3 | $\cdot 3$ | $\cdot 4$ | 4 | － 5 | 5 | 6 |
| Dec． | 0.3 | 0.3 | 0.2 | 0.2 | 0.2 | 0.2 | $0 \cdot 2$ | 0.2 | 0.3 | 0.3 | 0.4 | 0.4 |


| Lat．AZIMUTH |  |  |  |  |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| － | － | 。 | － | － | － | － | － | － |  | － |  |  |
| 0 | $359 \cdot 3$ | $359 \cdot 3$ | $359 \cdot 3$ | $359 \cdot 4$ | $359 \cdot 4$ | $359 \cdot 5$ | 359.6 | $359 \cdot 7$ | $359 \cdot 8$ | $359 \cdot 9$ | 0.0 | $0 \cdot 1$ |
| 20 | $359 \cdot 3$ | $359 \cdot 3$ | $359 \cdot 3$ | $359 \cdot 3$ | 359.4 | 359.5 | 359.6 | 3597 | $359 \cdot 8$ | 359.9 | 0.0 | $0 \cdot 1$ |
| 40 | 359.1 | $359 \cdot 1$ | 359. I | $359 \cdot 2$ | 359＊3 | 359.4 | 359.5 | 359.6 | 3597 | 359.9 | 0.0 | 0.2 |
| 50 | 359.0 | 359.0 | 359.0 | 359.0 | 359．1 | $359 \cdot 2$ | 359.4 | 359.5 | 3597 | 359.9 | $0 \cdot 0$ | 0.2 |
| 55 | 358.8 | 358.8 | 358.9 | 358.9 | 359.0 | 359.1 | $359 \cdot 3$ | 359.5 | 359.6 | $359 \cdot 8$ | $0 \cdot 0$ | 0.2 |
| 60 | $358 \cdot 7$ | $358 \cdot 7$ | $358 \cdot 7$ | $358 \cdot 8$ | 358.9 | 359.0 | 359.2 | 359.4 | 359.6 | $359 \cdot 8$ | 0.0 | $0 \cdot 3$ |
| 65 | 358.4 | 358.4 | 358.5 | 358.5 | 358.7 | 358.8 | 359.0 | 359.3 | 359.5 | $359 \cdot 8$ | 0.0 | $0 \cdot 3$ |
| ILLUSTRATION |  |  |  | From the daily pages： |  |  | －， |  | $H_{0}$ |  |  | 4931.6 |
| On 2016 April 21 at |  |  |  | GHA Aries（ $23^{\text {h }}$ ） |  |  | $195 \quad 26 \cdot 5$ |  | $a_{0}$（argument $162^{\circ} 57^{\prime}$ ） |  |  | 118.9 |
|  |  |  |  | Increment（ $18{ }^{\text {m }} 56^{6}$ ） |  |  | 444.8 |  | $a_{1}$（Lat $50^{\circ}$ approx．） |  |  | 0.6 |
| W $37^{\circ} 14^{\prime}$ the apparent altitude |  |  |  | Longitude（west） |  |  | －37 14 |  | $a_{2}$（April） |  |  | 0.9 |
| Polaris | was $49^{\circ}$ |  |  | LHA Aries |  |  | 16257 |  | Sum $-\mathrm{r}^{\circ}=$ Lat $=$ |  |  | 4952.0 |

Figure 1912b．Excerpt from the Polaris Tables．
calculator．See Figure 1913 for an example of a typical day＇s celestial plot．

1．Before dawn，compute the time of morning twilight and plot the dead reckoning position for that time．（See


Figure 1913. Typical celestial plot at sea.

Chapter 21 The Almanacs - Section 2109 Finding Times of Sunrise and Sunset.)
2. At morning twilight, take and reduce celestial observations for a fix. At sunrise take an amplitude of the Sun to obtain gyro error. (See Chapter 15 Azimuths and Amplitudes.)
3. Mid-morning, compare the chronometer with UT to determine chronometer error using a radio time tick. (See Chapter 20 Time.)
4. Mid-morning, reduce a Sun sight for a morning Sun line. (See section 1907 in this chapter.)
5. Calculate an azimuth of the Sun to obtain gyro error, if no amplitude was taken at sunrise.(See Chapter 15 Azimuths and Amplitudes.)
6. At LAN, obtain a Sun line and advance the morning Sun line for the noon fix. Compute a longitude determined at LAN for an additional LOP. (See Section

1910 and Section 1911 in this chapter.)
7. Mid-afternoon, again take and reduce a Sun sight. This is primarily for use with an advanced noon Sun line, or with a Moon or Venus line if the skies are overcast during evening twilight.
8. Calculate an azimuth of the Sun to obtain gyro error at about the same time as the afternoon Sun observation. The navigator may replace this azimuth with an amplitude observation at sunset. (See Chapter 15 Azimuths and Amplitudes.)
9. During evening twilight, reduce celestial observations for a fix. (See Section 1906 in this chapter.)
10. Be alert at all times for the moon or brighter planets which may be visible during daylight hours for additional LOP's, and Polaris at twilight for a latitude line.

