# CHAPTER 17 

## THE ALMANACS

## PURPOSE OF ALMANACS

## 1700. Introduction

Celestial navigation requires accurate predictions of the geographic positions of the celestial bodies observed. These predictions are available from three almanacs published annually by the U.S. Naval Observatory and H.M. Nautical Almanac Office (part of the U.K. Hydrographic Office) in England.

The Astronomical Almanac precisely tabulates celestial data for the exacting requirements found in scientific fields. Its precision is far greater than that required by celestial navigation. Even if the Astronomical Almanac is used for celestial navigation, it will not necessarily result in more accurate fixes due to the limitations of other aspects of the celestial navigation process. This printed book is available in the U.S. through the Government Publishing Office eBookstore and resellers, and elsewhere via U.K. Hydrographic Office distributors. There is also an Astronomical Almanac Online complementary website.

The Nautical Almanac contains astronomical information specifically needed by marine navigators. Information is tabulated to the nearest $0.1^{\prime}$ of arc and, with interpolation, to 1 second of time. GHA and declination are available for the Sun, Moon, planets (Venus, Mars, Jupiter and Saturn), and 173 stars, as well as corrections necessary to reduce the observed values to true. Also included are Sun rise/set, equation of time, Moonrise/set, moon phase, twilight times, time zones, and star charts. Explanations, examples, and sight reduction procedures are also given. This printed book is available in the U.S. through the Government Publishing Office eBookstore and resellers, and elsewhere via U.K. Hydrographic Office distributors.

The Air Almanac was originally intended for air navigators, but is used today mostly by the maritime community. In general, the information is similar to the Nautical Almanac, but is given to a precision of 1' of arc and 1 second of time, at intervals of 10 minutes (values for the Sun and Aries are given to a precision of 0.1'). Unique to the Air Almanac are its monthly sky diagrams, used to find navigational stars, planets, Sun and Moon at various
latitudes. This publication is suitable for ordinary navigation at sea, but lacks the precision of the Nautical Almanac, and provides GHA and declination for only the 57 commonly used navigation stars. The Air Almanac is available on CD or as a free download through the Government Publishing Office eBookstore, The CD and download contain the same information as was found previously in the annual publications, with page images in PDF files.

The US Naval Observatory also provides celestial navigation data via the web at http://aa.usno.navy.mil/data/.


Figure 1700. USNO Data Services.
This robust website includes a navigational star chart and other data services, which provide GHA, declination, computed altitude, azimuth and altitude correction information for the navigational objects above the horizon at a given assumed position and time. Additional data services found on this website includes Rise/Set/Transit/Twilight data, Phases of the Moon, Eclipses and Transits, Positions of Selected Celestial Bodies, Synthetic Views of Selected Solar System Bodies and Dates \& Times.

The Navy's STELLA program (System To Estimate Latitude and Longitude Astronomically), found aboard all seagoing Navy and Coast Guard vessels, contains an interactive almanac as well; this product is restricted to DoD and $\operatorname{DoD}$ contractors. A variety of privately produced electronic almanacs are available as computer programs or applications (apps). These invariably are associated with sight reduction software which replaces tabular and mathematical sight reduction methods.

## FORMAT OF THE NAUTICAL AND AIR ALMANACS

## 1701. Nautical Almanac

The major portion of the Nautical Almanac (pages 10
to 253 ) is devoted to hourly tabulations of Greenwich Hour Angle (GHA) and declination, to the nearest 0.1 ' of arc. On each set of facing pages, information is listed for
three consecutive days. On the left-hand page, successive columns list GHA of Aries ( $\wp$ ), and both GHA and declination of Venus, Mars, Jupiter, and Saturn, followed by the Sidereal Hour Angle (SHA) and declination of 57 stars. The GHA and declination of the Sun and Moon, and the horizontal parallax of the Moon, are listed on the righthand page. Where applicable, the quantities $v$ and d are given to assist in interpolation. The quantity $v$ is the difference between the actual change of GHA in 1 hour and a constant value used in the interpolation tables, while d is the change in declination in 1 hour. Both $v$ and $d$ are listed to the nearest $0.1^{\prime}$.

On the left hand pages, the magnitude of each planet at Universal Time (UT) 1200 of the middle day of the three listed on a given page is found at the top of the column. The UT of transit across the Greenwich meridian is listed as "Mer. Pass.". The value for the first point of Aries for the middle of the three days is listed to the nearest 0.1 ' at the bottom of the Aries column. The time of transit of the planets for the middle day is given to the nearest whole minute, with SHA (at UT 0000 of the middle day) to the nearest 0.1 ', below the list of stars.

On the right hand pages, to the right of the Moon data is listed the Local Mean Time (LMT) of sunrise, sunset, and beginning and ending of nautical and civil twilight for latitudes from $72^{\circ} \mathrm{N}$ to $60^{\circ} \mathrm{S}$. These times, which are given to the nearest minute, are UT of the phenomena on the Greenwich meridian. They are given for every day for moonrise and moonset, but only for the middle day of the three on each page for solar phenomena. For the Sun and Moon, the time of transit to the nearest whole minute is given for each day. For the Moon, both upper and lower transits are given. Also listed, are the equation of time for $0^{\mathrm{h}}$ and $12^{\mathrm{h}}$ UT, without sign, to the nearest whole second, with negative values shaded. The age and phase of the Moon is listed; age is given to the nearest whole day and phase is given by symbol. The semidiameters of both the Sun and Moon are also listed.

The main tabulation is preceded by a list of religious and civil holidays, phases of the Moon, a calendar, information on eclipses occurring during the year, and notes and a diagram giving information on the planets.

The main tabulation is followed by explanations and examples (pages 254 to 261). Next are four pages of standard times (zone descriptions). Star charts are next (pages 266267), followed by a list of 173 stars in order of increasing SHA. This list includes the 57 stars given on the daily pages, identified by a number in the "Name and Number" field. It gives the SHA and declination each month, and the magnitude.

Stars are listed by Bayer's name, a designation that originated from Johann Bayer, a German uranographer (celestial cartographer), who in 1603 published an atlas that named the entire celestial sphere. The Bayer's name is used to identify these stars and also the popular name is listed where applicable. The brightest stars have been given a
designation consisting of a Greek letter followed by the possessive form of the name of the constellation to which they belong.

Following the star list are the Polaris tables (pages 274276). These tables give the azimuth and the corrections to be applied to the observed Polaris altitude to find one's latitude.

Following the Polaris table is the "Sight Reduction Procedures" section, divided into two subsections. The first, "Methods and Formula for Direct Computation" (pages 277 to 283) gives formulas and examples for the entry of almanac data, the calculations that reduce a sight, and a method of solution for position, all for use with a calculator or computer. The second, "Use of Concise Sight Reduction Tables" (pages 284 to 319), gives instructions and examples of how to use the provided concise sight reduction tables and a sight reduction form. Tabular precision of the concise tables is one minute of arc.

The next pages (pp. 320-325) contain data on polar phenomena. Examples and graphs are given to estimate times of sunrise, sunset, civil twilight, moonrise, and moonset at extreme northern latitudes for each month of the year.

Next is a table for converting arc to time units (page i). This is followed by a 30-page table (pages ii - xxxi) called "Increments and Corrections," used for interpolation of the hourly GHA and declination to get to the nearest second of the sextant observation. This table is printed on tinted paper for quick location. Then come tables for interpolating for times of rise, set, and twilight (page xxxii); followed by two indices of the 57 stars listed on the daily pages, one index in alphabetical order, and the other in order of decreasing SHA (page xxxiii).

Altitude corrections are given at the front and back of the almanac. Tables for the Sun, stars, and planets, and a dip table, are given on the inside front cover and facing page, with an additional correction for nonstandard temperature and atmospheric pressure on the following page. Tables for the Moon, and an abbreviated dip table, are given on the inside back cover and facing page. Corrections for the Sun, stars, and planets for altitudes greater than $10^{\circ}$, and the dip table, are repeated on one side of a loose bookmark. The star indices are repeated on the other side.

## 1702. Air Almanac

The Air Almanac used to be a printed publication, but this was discontinued after the 2007 edition. Since then it has been available as a CD-ROM, and recently as a free download from the US Naval Observatory website: http://aa.usno.navy.mil/publications and the Government Publishing Office website. The electronic version contains the same information as was previously found in the printed version, but with PDFs of the page images. Navigation through the e-book is done via a web interface and two options are given. The default option is a "logical" layout and a second is a "book
layout", which maintains the same page order as the printed book. The description below follows the book layout.

As in the Nautical Almanac, the major portion of the Air Almanac is devoted to a tabulation of GHA and declination. However, in the Air Almanac values are listed at intervals of 10 minutes, to a precision of 0.1 ' for the Sun and Aries, and to a precision of 1 ' for the Moon and the planets. Values are given for the Sun, first point of Aries (GHA only), the three navigational planets most favorably located for observation, and the Moon. The magnitude of each planet listed is given at the top of its column, and the percentage of the Moon's disc illuminated, waxing (+) or waning $(-)$, is given at the bottom of each page. The magnitude of each planet listed is given at the top of its column. Values for the first 12 hours of the day are given on the right-hand page, and those for the second half of the day on the left-hand page. Each daily page includes the UT of moonrise and moonset on the Greenwich meridian for latitudes from $72^{\circ} \mathrm{N}$ to $60^{\circ} \mathrm{S}$; a "half-day" difference column provides data to find the time of moonrise and moonset at any longitude. In addition, each page has a critical table of the Moon's parallax in altitude, and below this the semidiameter of the Sun and Moon, and the percentage of the Moon's disc illuminated and whether it is waxing ( + ) or waning (-).

Critical tables for interpolation for GHA are given on
the inside front cover, which also has an alphabetical listing of the 57 navigational stars, with the number, magnitude, yearly mean SHA, and yearly mean declination of each. The same interpolation table and star list are printed on a flap which follows the daily pages. This flap also contains a star chart, a star list with the same data as the other, but in increasing navigational number order, and a table for interpolation of the UT of moonrise and moonset for longitude.

Following the flap are instructions for the use of the almanac; a list of symbols and abbreviations in English, French, and Spanish; a list of time differences between Greenwich and other places; monthly sky diagrams by latitude and time of day; planet location diagrams; star recognition diagrams for periscopic sextants; sunrise, sunset, and civil twilight tables; rising, setting, and depression graphs; semiduration graphs of Sunlight, twilight, and Moonlight in high latitudes; a single Polaris correction table; a list of 173 stars by number and Bayer designation (also popular name where there is one), giving the SHA and declination each month (to a precision of 0.1 '), and the magnitude; tables for interpolation of GHA Sun and GHA $\Upsilon$; a table for converting arc to time; a refraction correction table; a Coriolis correction table; and on the inside back cover, an aircraft standard dome refraction table; a correction table for dip of the horizon.

## USING THE ALMANACS

## 1703. Entering Arguments

The time used as an entering argument in the almanacs is UT, (formerly referred to as GMT), which is equivalent to $12^{\mathrm{h}}+$ GHA of the mean Sun. This scale may differ from the broadcast time signals by an amount of $0.9^{\mathrm{s}}$ which, if ignored, will introduce an error of up to $0.2^{\prime}$ in longitude determined from astronomical observations. The difference arises because the time argument depends on the variable rate of rotation of the Earth while the broadcast time signals are now based on atomic time. Leap seconds, that is step adjustments of exactly one second are made to the time signals as required (primarily at 24 h on December 31 and June 30) so that the difference between the time signals and UT, as used in the almanacs, may not exceed $0.9^{\text {s }}$. If observations to a precision of better than $1^{\mathrm{s}}$ are required, corrections must be obtained from coding in the signal, or from other sources. The correction may be applied to each of the times of observation. Alternatively, the longitude, when determined from observations, may be corrected by the corresponding amount shown in Table 1703.

The main contents of the almanacs consist of data from which the GHA and the declination of all the bodies used for navigation can be obtained for any instant of UT. The

| Correction to time <br> signals | Correction to <br> longitude |
| :---: | :---: |
| $-0.9^{\mathrm{s}}$ to $-0.7^{\mathrm{s}}$ | $0.2^{\prime}$ to east |
| $-0.6^{\mathrm{s}}$ to $-0.3^{\mathrm{s}}$ | $0.1^{\prime}$ to east |
| $-0.2^{\mathrm{s}}$ to $+0.2^{\mathrm{s}}$ | no correction |
| $+0.3^{\mathrm{s}}$ to $+0.6^{\mathrm{s}}$ | 0.1 ' to west |
| $+0.7^{\mathrm{s}}$ to $+0.9^{\mathrm{s}}$ | 0.2 ' to west |

Table 1703. Corrections to time.
LHA can then be obtained with the formula:

$$
\begin{aligned}
\text { LHA } & =\text { GHA }+ \text { east longitude } \\
\text { LHA } & =\text { GHA }- \text { west longitude }
\end{aligned}
$$

For the Sun, Moon, and the four navigational planets, the GHA and declination are tabulated directly in the Nautical Almanac for each hour of UT throughout the year; in the Air Almanac, the values are tabulated for each whole 10 m of UT. For the stars, the SHA is given, and the GHA is obtained from:

GHA Star $=$ GHA $\Upsilon+$ SHA Star.

The SHA and declination of the stars change slowly and may be regarded as constant over periods of several days or even months if lesser accuracy is required. The SHA and declination of stars tabulated in the Air Almanac may be considered constant to a precision of 1.5 ' to 2 ' for the period covered by each of the volumes providing the data for a whole year, with most data being closer to the smaller value. GHA $\Upsilon$, or the GHA of the first point of Aries (the vernal equinox), is tabulated for each hour in the Nautical Almanac and for each whole $10^{\mathrm{m}}$ in the Air Almanac. Permanent tables list the appropriate increments to the tabulated values of GHA and declination for the minutes and seconds of time.

In the Nautical Almanac, the permanent table for increments also includes corrections for $v$, the difference between the actual change of GHA in one hour and a constant value used in the interpolation tables; and $d$, the average hourly change in declination.

In the Nautical Almanac, $v$ is always positive unless a negative sign (-) is shown. This occurs only in the case of Venus. For the Sun, the tabulated values of GHA have been adjusted to reduce to a minimum the error caused by treating $v$ as negligible; there is no $v$ tabulated for the Sun.

No sign is given for tabulated values of $d$; whether to add or subtract a correction to the declination must be done via inspection of the increasing or decreasing trend of the declination values.

In the Air Almanac, the tabulated declination values, except for the Sun, are those for the middle of the interval between the time indicated and the next following time for which a value is given, making interpolation unnecessary. Thus, it is always important to take out the GHA and declination for the time immediately before the time of observation.

In the Air Almanac, GHA $\Upsilon$ and the GHA and declination of the Sun are tabulated to a precision of $0.1^{\prime}$. If these values are extracted with the tabular precision, the "Interpolation of GHA" table on the inside front cover (and flap) should not be used; use the "Interpolation of GHA Sun" and "Interpolation of GHA Aries' tables, as appropriate. These tables are found on pages A164 and A165.

## 1704. Finding GHA and Declination of the Sun

Nautical Almanac: Enter the daily page table with the whole hour before the given GMT, unless the exact time is a whole hour, and take out the tabulated GHA and declination. Inspect the trend in the following declination value to determine if declination is increasing or decreasing; this is needed to know whether to add or subtract the $d$ correction. Also record the $d$ value given at the bottom of the declination column. Next, enter the increments and corrections table for the number of minutes of GMT. If there are seconds, use the next earlier whole minute. On the line corresponding to the seconds of GMT, extract the value from the Sun-Planets column. Add this to
the value of GHA from the daily page. This is GHA of the Sun. Next, enter the correction table for the same minute of GMT with the $d$ value and take out the correction. Apply the $d$ correction, either adding or subtracting (as determined earlier by inspection of the tabulated declination values), to the declination from the daily page. This is the declination.

The correction table for GHA of the Sun is based upon a rate of change of $15^{\circ}$ per hour, the average rate during a year. At most times the rate differs slightly. Th e slight error is minimized by adjustment of the tabular values. The $d$ value is the average hourly amount that the declination changes on the middle day of the three shown.

Air Almanac: Enter the daily page with the whole $10^{\mathrm{m}}$ preceding the given GMT, unless the time is itself a whole $10^{\mathrm{m}}$, and extract the GHA. The declination is extracted without interpolation from the same line as the tabulated GHA or, in the case of planets, the top line of the block of six. If the values extracted are rounded to the nearest minute, enter the "Interpolation of GHA" table on the inside front cover (and flap), using the "Sun, etc." entry column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction listed half a line above the entry time. Add this correction to the GHA taken from the daily page. This is GHA. No adjustment of declination is needed. If the values are extracted with a precision of $0.1^{\prime}$, the table for interpolating the GHA of the Sun to a precision of 0.1' must be used (page A164). Again no adjustment of declination is needed.

## 1705. Finding GHA and Declination of the Moon

Nautical Almanac: Enter the daily page table with the whole hour before the given GMT, unless this time is itself a whole hour, and extract the tabulated GHA and declination. Record the corresponding $v$ and $d$ values tabulated on the same line, and determine whether the $d$ correction is to be added or subtracted, by inspecting the trend in the next tabular declination value. The $v$ value of the Moon is always positive $(+)$ but it is not marked in the almanac. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT, take the GHA correction from the Moon column. Then, enter the correction table for the same minute with the $v$ value, and extract the correction. Add both of these corrections to the GHA from the daily page. This is the GHA of the Moon. Then, enter the same correction table page with the $d$ value and extract the corresponding d correction. Apply the $d$ correction, either adding or subtracting (as determined earlier by inspection of the trend of the tabulated declination values), to the declination from the daily page. This is the declination of the Moon.

The correction table for GHA of the Moon is based upon the minimum rate at which the Moon's GHA increases, $14^{\circ} 19.0^{\prime}$ per hour. The $v$ correction adjusts for the actual rate. The $v$ value is the difference between the
minimum rate and the actual rate during the hour following the tabulated time. The $d$ value is the amount that the declination changes during the hour following the tabulated time.

Air Almanac: Enter the daily page with the whole 10 m next preceding the given GMT, unless this time is a whole 10 m , and extract the tabulated GHA and the declination without interpolation. Next, enter the "Interpolation of GHA" table on the inside front cover, using the "Moon" entry column, and extract the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line above the entry time. Add this correction to the GHA taken from the daily page to find the GHA at the given time. No adjustment of declination is needed.

The declination given in the table is correct for the time 5 minutes later than tabulated, so that it can be used for the 10minute interval without interpolation, to an accuracy to meet most requirements. Declination changes much more slowly than GHA. If greater accuracy is needed, it can be obtained by interpolation, remembering to allow for the 5 minutes.

## 1706. Finding GHA and Declination of a Planet

Nautical Almanac: Enter the daily page table with the whole hour before the given GMT, unless the time is a whole hour, and extract the tabulated GHA and declination. Record the $v$ and $d$ values given at the bottom of each of these columns; determine whether the $d$ correction is to be added or subtracted by inspecting the trend in the declination. Next, enter the increments and corrections table for the minutes of GMT, and on the line for the seconds of GMT, take the GHA correction from the Sunplanets column. Next, enter the correction table with the $v$ value and extract the correction, giving it the sign of the $v$ value. Add the first correction to the GHA from the daily page, and apply the second correction in accordance with its sign. This is the GHA of the planet. Then enter the increments and correction table for the same minute with the $d$ value, and extract the correction. Apply the $d$ correction, either adding or subtracting (as determined earlier by inspection of the tabulated declination values), to the declination from the daily page to find the declination of the planet at the given time.

The correction table for GHA of planets is based upon the mean rate of the Sun, $15^{\circ}$ per hour. The $v$ value is the difference between $15^{\circ}$ and the average hourly change of GHA of the planet on the middle day of the three shown. The $d$ value is the average hourly amount the declination changes on the middle day. Venus is the only body listed which ever has a negative $v$ value.

Air Almanac: Enter the daily page with the whole $10^{\mathrm{m}}$ before the given GMT, unless this time is a whole 10 m , and extract the tabulated GHA and declination, without interpolation. The tabulated declination is correct for the time $30^{\mathrm{m}}$ later than tabulated, so interpolation during the hour
following tabulation is not needed for most purposes. Next, enter the "Interpolation of GHA" table on the inside front cover, using the "Sun, etc." column, and take out the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction half a line above the entry time. Add this correction to the GHA from the daily page to find the GHA at the given time. No adjustment of declination is needed.

## 1707. Finding GHA and Declination of a Star

If the GHA and declination of each navigational star were tabulated separately, the almanacs would be several times their present size. But since the sidereal hour angle and the declination are nearly constant over several days (to the nearest 0.1 ') or months (to the nearest 1 '), separate tabulations are not needed. Instead, the GHA of the first point of Aries, from which SHA is measured, is tabulated on the daily pages, In the Nautical Almanac, a single listing of SHA and declination for the 57 navigational stars is given for each double page (computed at 12 UT1 of the middle day); monthly values are given for 173 bright stars (pages 268 through 273). In the Air Almanac, the yearly mean SHA and declinations are listed on the inside cover and flap; for higher accuracy, monthly values are tabulated for the 173 navigation stars (pages A158 through A163). Finding the GHA is similar to finding the GHA of the Sun, Moon, and planets.

Nautical Almanac: Enter the daily page table with the whole hour before the given GMT, unless this time is a whole hour, and extract the tabulated GHA of Aries. Also record the tabulated SHA and declination of the star from the listing on the left-hand daily page. Next, enter the increments and corrections table for the minutes of GMT, and, on the line for the seconds of GMT, extract the GHA correction from the Aries column. Add this correction and the SHA of the star to the GHA $\gamma$ on the daily page to find the GHA of the star at the given time. Subtraction of $360^{\circ}$ may be necessary to keep GHA between $0^{\circ}$ and $360^{\circ}$. No adjustment of declination is needed.

The SHA and declination of 173 stars, including Polaris and the 57 listed on the daily pages, are given for the middle of each month. For a star not listed on the daily pages, this is the only almanac source of this information. Interpolation in this table is not necessary for ordinary purposes of navigation, but is sometimes needed for precise results.

Air Almanac: Enter the daily page with the whole 10 m before the given GMT, unless this is a whole $10^{\mathrm{m}}$, and extract the tabulated GHA $\Upsilon$. Next, enter the "Interpolation of GHA" table on the inside front cover, using the "Sun, etc." entry column, and extract the value for the remaining minutes and seconds of GMT. If the entry time is an exact tabulated value, use the correction given half a line above the entry time. From the tabulation at the left side of the same page, extract the SHA and declination of the star. Add the GHA from the daily page and the two values
taken from the inside front cover to find the GHA at the given time. No adjustment of declination is needed. Should higher precision be needed, use the SHA and declination
values on pages A158 to A163, and the interpolation of GHA Aries table on A165.

## RISING, SETTING, AND TWILIGHT

## 1708. Rising, Setting, and Twilight

In both Air and Nautical Almanacs, the times of sunrise, sunset, moonrise, moonset, and twilight information, at various latitudes between $72^{\circ} \mathrm{N}$ and $60^{\circ} \mathrm{S}$, is listed to the nearest whole minute. By definition, rising or setting occurs when the upper limb of the body is on the visible horizon, assuming standard refraction for zero height of eye. Because of variations in refraction and height of eye, computation to a greater precision than 1 minute of time is not justified.

In high latitudes, some of the phenomena do not occur during certain periods. Symbols are used in the almanacs to indicate:

1. Sun or Moon does not set, but remains continuously above the horizon, indicated by an open rectangle.
2. Sun or Moon does not rise, but remains continuously below the horizon, indicated by a solid rectangle.
3. Twilight lasts all night, indicated by 4 slashes (////).

Both the Nautical Almanac and the Air Almanac provide graphs for finding the times of rising, setting, or twilight in polar regions.

In the Nautical Almanac, sunrise, sunset, and twilight tables are given only once for the middle of the three days on each page opening. Moonrise and moonset tables are given for each day. For many purposes this information can be used for all three days. For high precision needs, interpolation tables are provided (page xxxii). In the Air Almanac, sunrise, sunset, and twilight tables are given every three days (pages A130-A145). Graphs and tables are provided to compute phenomena at altitudes up to 60,000 feet. Moonrise and moonset tables are given daily in the main table.

The tabulations are in UT on the Greenwich meridian. They are approximately the LMT of the corresponding phenomena on other meridians; they can be formally interpolated if desired. The conversion of UT to LMT and vice versa of a phenomenon is obtained by the formula:

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UT = LMT + W Longitude
UT = LMT - E Longitude
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To use this formula, convert the longitude to time using the table on page $i$ or by computation, and add or subtract as indicated.

## 1709. Finding Times of Sunrise and Sunset

To find the time of sunrise or sunset in the Nautical

Almanac, enter the table on the daily page, and extract the LMT for the latitude next smaller than your own (unless it is exactly the same). Apply a correction from Table I on almanac page xxxii to interpolate for latitude, determining the sign by inspection. Then convert LMT to ZT using the difference of longitude between the local and zone meridians.

For the Air Almanac, the procedure is the same as for the Nautical Almanac, except that the LMT is taken from the tables of sunrise and sunset instead of from the daily page, and the latitude correction is by linear interpolation.

The tabulated times are for the Greenwich meridian. Except in high latitudes near the time of the equinoxes, the time of sunrise and sunset varies so little from day to day that no interpolation is needed for longitude. In high latitudes interpolation is not always possible. Between two tabulated entries, the Sun may in fact cease to set. In this case, the time of rising and setting is greatly influenced by small variations in refraction and changes in height of eye.

## 1710. Twilight

Morning twilight ends at sunrise, and evening twilight begins at sunset. The time of the darker limit can be found from the almanacs. The time of the darker limits of both civil and nautical twilights (center of the Sun $6^{\circ}$ and $12^{\circ}$, respectively, below the celestial horizon) is given in the Nautical Almanac. The Air Almanac provides tabulations of civil twilight from $60^{\circ} \mathrm{S}$ to $72^{\circ} \mathrm{N}$. The brightness of the sky at any given depression of the Sun below the horizon may vary considerably from day to day, depending upon the amount of cloudiness, haze, and other atmospheric conditions. In general, the most effective period for observing stars and planets occurs when the center of the Sun is between about $3^{\circ}$ and $9^{\circ}$ below the celestial horizon. Hence, the darker limit of civil twilight occurs at about the mid-point of this period. At the darker limit of nautical twilight, the horizon is generally too dark for good observations.

At the darker limit of astronomical twilight (center of the Sun $18^{\circ}$ below the celestial horizon), full night has set in. The time of this twilight is given in the Astronomical Almanac. Its approximate value can be determined by extrapolation in the Nautical Almanac, noting that the duration of the different kinds of twilight is proportional to the number of degrees of depression for the center of the Sun. More precise determination of the time at which the center of the Sun is any given number of degrees below the celestial horizon can be determined by a large-scale diagram on the plane of the celestial meridian, or by
computation. Duration of twilight in latitudes higher than $65^{\circ} \mathrm{N}$ is given in a graph in both the Nautical and the Air Almanac.

In both Nautical and Air Almanacs, the method of finding the darker limit of twilight is the same as that for sunrise and sunset.

Sometimes in high latitudes the Sun does not rise but twilight occurs. This is indicated in the almanacs by a solid black rectangle symbol in the sunrise and sunset column. To find the time of beginning of morning twilight, subtract half the duration of twilight as obtained from the duration of twilight graph from the time of meridian transit of the Sun; and for the time of ending of evening twilight, add it to the time of meridian transit. The LMT of meridian transit never differs by more than $16.4^{\mathrm{m}}$ (approximately) from 1200. The actual time on any date can be determined from the almanac.

## 1711. Moonrise and Moonset

Finding the time of moonrise and moonset is similar to finding the time of sunrise and sunset, with one important difference. Because of the Moon's rapid change of declination, and its fast eastward motion relative to the Sun, the time of moonrise and moonset varies considerably from day to day. These changes of position on the celestial sphere are continuous and complex. For precise results, it would be necessary to compute the time of the phenomena at any given place by lengthy complex calculation. For ordinary purposes of navigation, however, it is sufficiently accurate to interpolate between consecutive moonrises or moonsets at the Greenwich meridian. Since apparent motion of the Moon is westward, relative to an observer on the Earth, interpolation in west longitude is between the phenomenon on the given date and the following one. In east longitude it is between the phenomenon on the given date and the preceding one.

To find the time of moonrise or moonset in the Nautical Almanac, enter the daily pages table with latitude and extract the LMT for the tabulated latitude next smaller than the observer's latitude (unless this is an exact tabulated value). Apply a correction from table I of almanac page xxxii to interpolate for latitude, determining the sign of the correction by inspection. Repeat this procedure for the day following the given date, if in west longitude; or for the day preceding, if in east longitude. Using the difference between these two times, and the longitude, enter table II of the almanac on the same page and take out the correction. Apply this correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The sign to be given the correction is such as to make the corrected time fall between the times for the two dates between which interpolation is being made. This is nearly always positive ( + ) in west longitude and negative ( - ) in east longitude. Convert the corrected LMT to ZT.

To find the time of moonrise or moonset by the Air

Almanac for the given date, determine LMT for the observer's latitude at the Greenwich meridian in the same manner as with the Nautical Almanac, except that linear interpolation is made directly from the main tables, since no interpolation table is provided. Extract, also, the value from the "Diff." column to the right of the moonrise and moonset column, interpolating if necessary. This "Diff." is the halfdaily difference. The error introduced by this approximation is generally not more than a few minutes, although it increases with latitude. Using this difference, and the longitude, enter the "Interpolation of moonrise, moonset" table on flap F4 of the Air Almanac and extract the correction. The Air Almanac recommends taking the correction from this table without interpolation. The results thus obtained are sufficiently accurate for ordinary purposes of navigation. If greater accuracy is desired, the correction can be taken by interpolation. However, since the "Diff." itself is an approximation, the Nautical Almanac or computation should be used if accuracy is a consideration. Apply the correction to the LMT of moonrise or moonset at the Greenwich meridian on the given date to find the LMT at the position of the observer. The correction is positive (+) for west longitude, and negative (-) for east longitude, unless the "Diff." on the daily page is preceded by the negative sign (-), when the correction is negative ( - ) for west longitude, and positive ( + ) for east longitude. If the time is near midnight, record the date at each step, as in the Nautical Almanac solution.

As with the Sun, there are times in high latitudes when interpolation is inaccurate or impossible. At such periods, the times of the phenomena themselves are uncertain, but an approximate answer can be obtained by the Moonlight graph in the almanacs. With the Moon, this condition occurs when the Moon rises or sets at one latitude, but not at the next higher tabulated latitude. It also occurs when the Moon rises or sets on one day, but not on the preceding or following day. This latter condition is indicated in the Air Almanac by the symbol * in the "Diff." column.

Because of the eastward revolution of the Moon around the Earth, there is one day each synodical month $\left(29^{1} / 2\right.$ days) when the Moon does not rise, and one day when it does not set. These occur near last quarter and first quarter, respectively. This day is not the same at all latitudes or at all longitudes, thus the time of moonrise or moonset found from the almanac may occasionally be the preceding or succeeding one to that desired (indicated by a time greater than $23^{\mathrm{h}} 59^{\mathrm{m}}$ ). When interpolating near midnight, caution will prevent an error.

The effect of the revolution of the Moon around the Earth, generally, is to cause the Moon to rise or set later from day to day. The daily retardation due to this effect does not differ greatly from $50^{\mathrm{m}}$. However, the change in declination of the Moon may increase or decrease this effect. This effect increases with latitude, and in extreme conditions it may be greater than the effect due to revolution of the Moon. Hence, the interval between
successive moonrises or moonsets is more erratic in high latitudes than in low latitudes. When the two effects act in the same direction, daily differences can be quite large. When they act in opposite directions, they are small, and when the effect due to change in declination is larger than that due to revolution, the Moon sets earlier on succeeding days.

This condition is reflected in the Air Almanac by a negative "Diff." If this happens near the last quarter or first quarter, two moonrises or moonsets might occur on the same day, one a few minutes after the day begins, and the other a few minutes before it ends. Interpolation for longitude is always made between consecutive moonrises or moonsets, regardless of the days on which they fall.

Beyond the northern limits of the almanacs the values can be obtained from a series of graphs given near the back of the books (pages 322-325 for Nautical, A153-A157 for

Air). For high latitudes, graphs are used instead of tables because graphs give a clearer picture of conditions, which may change radically with relatively little change in position or date. Under these conditions interpolation to practical precision is simpler by graph than by table. In those parts of the graph which are difficult to read, the times of the phenomena's occurrence are uncertain, being altered considerably by a relatively small change in refraction or height of eye.

On all of these graphs, any given latitude is represented by a horizontal line and any given date by a vertical line. At the intersection of these two lines the duration is read from the curves, interpolating by eye between curves; see Figure 1711a for an example of a Semiduration of Moonlight plot for the month of January 2016.

## SEMIDURATION OF MOONLIGHT 2016



Figure 1711a. Semiduration of moonlight for high latitudes in January 2016.

The "Semiduration of Sunlight" graph gives the number of hours between sunrise and meridian transit or between meridian transit and sunset. The dot scale near the top of the graph indicates the LMT of meridian transit, the time represented by the minute dot nearest the vertical dateline being used. If the intersection occurs in the area marked "Sun above horizon," the Sun does not set; and if in the area marked "Sun below horizon," the Sun does not rise.

The "Duration of Twilight" graph gives the number of hours between the beginning of morning civil twilight (center of Sun $6^{\circ}$ below the horizon) and sunrise, or between sunset and the end of evening civil twilight. If the Sun does not rise, but twilight occurs, the time taken from the graph is half the total length of the single twilight period, or the number of hours from beginning of morning twilight to LAN, or from LAN to end of evening twilight. If the intersection occurs in the area marked "continuous twilight or Sunlight," the center of the Sun does not move
more than $6^{\circ}$ below the horizon, and if in the area marked "no twilight nor Sunlight," the Sun remains more than $6^{\circ}$ below the horizon throughout the entire day.

The "Semiduration of Moonlight" graph gives the number of hours between moonrise and meridian transit or between meridian transit and moonset. The dot near the top of the graph indicates the LMT of meridian passage, and the spacing between each dot is approximately 50 minutes. The phase symbols indicate the date on which the principal Moon phases occur, the open circle indicating full Moon and the dark circle indicating new Moon. If the intersection of the vertical dateline and the horizontal latitude line falls in the "Moon above horizon" or "Moon below horizon" area, the Moon remains above or below the horizon, respectively, for the entire 24 hours of the day.

If approximations of the times of moonrise and moonset are sufficient, the semiduration of Moonlight is taken for the time of meridian passage (dots along top scale) and can
be used without adjustment. For example, to estimate moonrise on 19 January 2016 at latitude $70^{\circ} \mathrm{N}$ and the following moonset, see Figure 1711b. Using the dot along the top scale, the semiduration of moonlight is 10 h at $70^{\circ} \mathrm{N}$. The meridian passage itself is about at 20:30 LMT, found by adding 50 minutes to each successive dot after the 18 h one. Approximate moonrise is the semiduration minus meridian passage, $10 \mathrm{~h}-20: 30$, or at 10:30 LMT. The following moonset is semiduration plus meridian passage,
$10 h+20: 30$, or at $06: 30$ the following day. For more accurate results (seldom justified), the times on the required date and the adjacent date (the following date in W longitude and the preceding date in E longitude) should be determined, and an interpolation made for longitude, as in any latitude, since the intervals given are for the Greenwich meridian.


Figure 1711b. Moon's meridian passage on 19 January 2016.

Sunlight, twilight and Moonlight graphs are not given for south latitudes. Beyond latitude $65^{\circ} \mathrm{S}$, the northern hemisphere graphs can be used for determining the semiduration or duration, by using the vertical dateline for a day when the declination has the same numerical value but opposite sign. The time of meridian transit and the phase of the Moon are determined as explained above, using the correct date. Between latitudes $60^{\circ} \mathrm{S}$ and $65^{\circ} \mathrm{S}$, the solution is made by interpolation between the tables and the graphs.

Semiduration or duration can be determined graphically using a diagram on the plane of the celestial meridian, or by computation. When computation is used, solution is made for the meridian angle at which the required negative altitude occurs. The meridian angle expressed in time units is the semiduration in the case of sunrise, sunset, moonrise, and moonset; and the semiduration of the combined Sunlight and twilight, or the time from meridian transit at which morning twilight begins or evening twilight ends. For sunrise and sunset the altitude used is (-)50'. Allowance for height of eye can be made by algebraically subtracting (numerically adding) the dip correction from this altitude. The altitude used for twilight is $(-) 6^{\circ},(-) 12^{\circ}$, or $(-) 18^{\circ}$ for civil,
nautical, or astronomical twilight, respectively. The altitude used for moonrise and moonset is -34' - SD + HP, where SD is semidiameter and HP is horizontal parallax, from the daily pages of the Nautical Almanac.

Other methods of solution of these phenomena are available. If an internet connection is available, the US Naval Observatory website provides calculators (aa.usno.navy.mil/data/). Sunrise and sunset for latitudes from $76^{\circ} \mathrm{N}$ to $60^{\circ} \mathrm{S}$ can be derived using Table 4 of NOAA's Tide Tables publications.

## 1712. Rising, Setting, and Twilight on a Moving Craft

Instructions to this point relate to a fixed position on the Earth. Aboard a moving craft the problem is complicated somewhat by the fact that time of occurrence depends upon the position of the craft, which itself depends on the time. The US military can use STELLA, which calculates phenomena from a moving platform (see Section 1900), for others, at ship speeds, it is generally sufficiently accurate to make an approximate mental solution and use the position of the vessel at this time to make a more
accurate solution. If greater accuracy is required, the position at the time indicated in the second solution can be used for a third solution. If desired, this process can be repeated until the same answer is obtained from two consecutive solutions. However, it is generally sufficient to alter the first solution by $1^{\mathrm{m}}$ for each 15 ' of longitude that
the position of the craft differs from that used in the solution, adding if west of the estimated position, and subtracting if east of it. In applying this rule, use both longitudes to the nearest $15^{\prime}$. The first solution is the first estimate; the second solution is the second estimate.

