

CHAPTER 16

TIME

TIME IN NAVIGATION

1600. Apparent and Solar Time

The Earth's rotation on its axis presents the Sun and other celestial bodies to appear to proceed across the sky from east to west each day. If a navigator measures the time interval between two successive transits across the local meridian of a very distant star by the passage of time against another physical time reference such as a chronometer, he or she would be measuring the period of the Earth's rotation.

In the most practical sense, the Earth's rotation is the navigator's standard of time. When the navigator then

makes a similar measurement of the transit of the Sun, the resulting time interval would be about four minutes longer than the period determined by the Earth's rotation. This is due to the Earth's yearly orbital motion around the Sun, which continuously changes the apparent position of the Sun against the background of stars, traditionally observed as the cyclical procession of the zodiac. Thus, during the course of a day, the Sun appears to lag a little to the east with respect to the stars, and the Earth's rotation must exceed a complete rotation (360°) in order to have the Sun appear overhead at the local meridian.

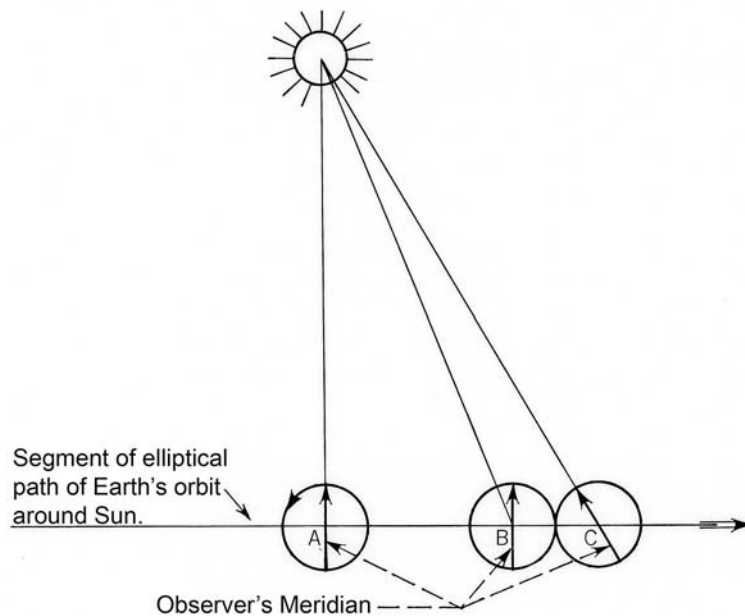


Figure 1600a. Apparent eastward movement of the Sun with respect to the stars.

Apparent eastward lag of the Sun with diurnal observations - when the Sun is on the observer's meridian at point A in the Earth's orbit around the Sun (see Figure 1600a), it will not be on the observer's meridian after the Earth has rotated once (360°) because the Earth will have moved along its orbit to point B. Before the Sun can again be observed on the observer's meridian, the Earth must turn a little more on its axis as shown in C. Thus, during the course of a day (as determined by the Earth's rotation period) the Sun appears to move eastward with respect to the celestial back-

ground of stars.

The apparent positions of individual stars against the celestial background are commonly determined with reference to an imaginary point called the **vernal equinox**. The vernal equinox is the intersection of the celestial equator and the ecliptic (see Figure 1600b). The full rotation of the Earth measured with respect to the vernal equinox is called a **sidereal day**, and corresponds to the Earth's rotational period. The period with respect to the Sun is called an **apparent solar day**, and includes the additional time to

compensate for the Earth's orbital motion.

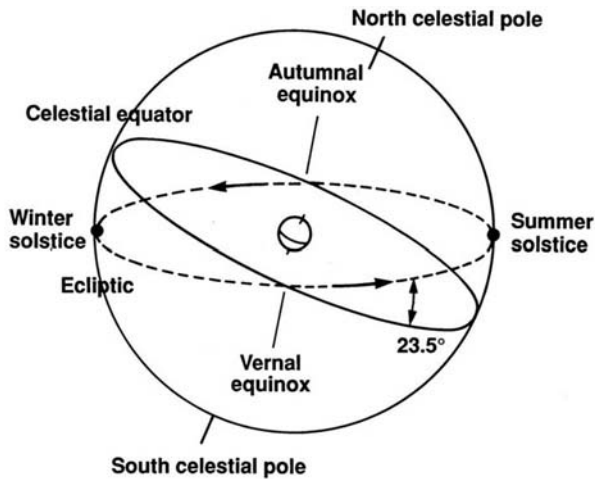


Figure 1600b. Solstices and equinoxes on the ecliptic.

A navigator using the observed position of the Sun, or the apparent Sun, to measure the passage of time from Earth's rotation results in **apparent solar time**. Apparent solar time is what a perfectly constructed and calibrated sundial would read at a given location, based on the apparent position of the Sun in the sky. In astronomical terms, apparent solar time is determined by the **local hour angle**, which is a measure of the Sun's projected angular distance east or west of the local **meridian**. Since each meridian is a line of constant longitude, at any instant of the Earth's rotation, the apparent solar time will differ for every longitude. We define apparent solar time at a specific location as 12^{h} + the local hour angle (expressed in hours) of the apparent position of the Sun in the sky. The local hour angle is negative when presenting east of the meridian.

Apparent solar time is not a uniform time scale; the apparent Sun crosses the sky at slightly different rates at different times of the year. This means the apparent solar time runs “fast” with respect to a constant timescale, such as a chronometer, part of the year and “slow” during other parts of the year. Although the daily fractional change in the rate of the Sun's apparent motion is small, the accumulated time difference can reach as much as sixteen minutes. This effect is a function of the Earth's orbit around the Sun. It is the result of two superimposed cycles; the Earth's eccentricity (no-circular orbit) and the tilt of Earth's axis with respect to the plane of its orbit (the ecliptic).

In order to create a uniform time scale for practical use, we imagine a point in the sky called the **fictitious mean sun**, which moves at a constant rate across the sky (at the celestial equator), regardless of the time of year. That is, the fictitious mean sun averages out the variations in the position and rate of motion of the true Sun over the course of an entire year. The fictitious mean sun is never more than about 4 degrees east or west of the actual Sun, although it is only an imaginary point. We can define **mean solar time** in

the same way as apparent solar time: mean solar time at a specific location is 12^{h} + the local hour angle (expressed in hours) of the fictitious mean sun. Of course, the fictitious mean sun is not an observable point, so we need a mathematical expression to tell us where it is with respect to the true Sun; this is the equation of time.

1601. Equation of Time

Mean solar time is sometimes ahead (fast) and sometimes behind (slow) of the apparent solar time. This difference is called the **equation of time**. The equation of time's minimum value is near -14 m 13 s in mid-February, and its maximum value is near 16 m 26 s in early November.

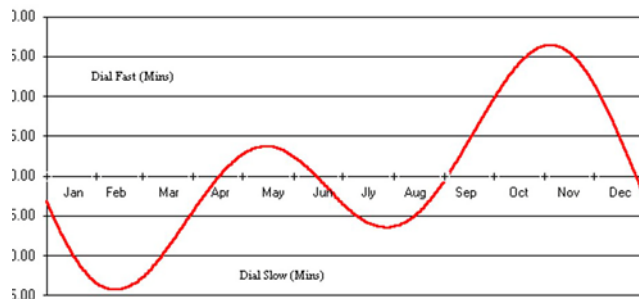


Figure 1601a. Equation of time.

The **equation of time** gives the offset in minutes applied to mean solar time, as may be determined by a chronometer, to calculate the apparent solar time; specifically at the Sun's apparent passage at the local meridian.

The navigator most often deals with the equation of time when determining the time of **upper meridian passage** of the Sun, called **Local Apparent Noon (LAN)**. Were it not for the difference in rate between the fictitious mean and apparent Sun, the Sun would always appear on the observer's meridian at 12^{h} (noon) local time. Except for four unique times of the year related to the interaction of the Earth's eccentric orbit and inclination to the ecliptic, the LAN will always be offset from exactly noon mean solar time. This time difference, which is applied as the equation of time at meridian transit, is listed on the right hand daily pages of the *Nautical Almanac*.

The sign of the equation of time is negative if apparent time is behind mean time; it is positive if apparent time is ahead of mean time. In either case, the equation is: $\text{Apparent Time} = \text{Mean Time} + (\text{equation of time})$. A negative equation of time is indicated by shading in the *Nautical Almanac*.

Example 1: Determine the local mean time of the Sun's meridian passage (Local Apparent Noon) on June 16, 2016.

Solution: See the *Nautical Almanac's* right hand daily page for June 16, 2016 (Figure 1601b). The equation of time is listed in the bottom right hand corner of the page.

There are two ways to solve the problem, depending on the accuracy required for the value of meridian passage. First, for minute accuracy, the time of the Sun at meridian passage is given to the nearest minute in the "Mer. Pass." column. For June 16, 2016, this time is 1201.

Second, to determine the nearest second of time of meridian passage, use the value given for the equation of time listed immediately to the left of the "Mer. Pass." column on the daily pages. For June 16, 2016, the value is given as negative 00 m 47 s. (Use the "12 h" column because the problem asked for meridian passage at LAN.)

Using the equation $Apparent\ Time = Mean\ Time + (equation\ of\ time)$, we have $1200 = Mean\ Time + (-0047)$. Rearranging, we get $Mean\ Time = 1200 + 0047$. The exact mean time of meridian passage for June 16, 2016, is 12 h 00 m 47 s.

To calculate latitude and longitude at LAN, the navigator seldom requires the time of meridian passage to accuracies greater than one minute (0.25 degrees of longitude). Therefore, use the time listed under the "Mer. Pass."

| | | | | | | | | | |
|-----|-----|-----------------|-----------------|-------|------------|-------|-------|-------|----|
| 4.2 | S | 50 | 16 03 | 16 42 | 17 23 | 01 33 | 03 33 | 04 34 | |
| 4.2 | | 52 | 15 54 | 16 34 | 17 18 | | | 04 39 | |
| 4.2 | | 54 | 15 42 | 16 26 | 17 11 | | | 04 45 | |
| 4.2 | | 56 | 15 30 | 16 17 | 17 06 | | | 04 52 | |
| 4.2 | | 58 | 15 15 | 16 07 | 17 00 | | | 04 59 | |
| 4.2 | S | 60 | 14 58 | 15 55 | 16 52 | | | 05 07 | |
| SUN | | | | | | | | | |
| 4.3 | Day | Eqn. of Time | | Mer. | Mer. Pass. | | Age | Phase | |
| 4.3 | | 00 ^h | 12 ^h | Pass. | Upper | Lower | | | |
| 4.3 | d | m | s | m | s | h | m | d | % |
| 4.3 | 14 | 00 | 16 | 00 | 22 | 12 | 00 | 19 | 49 |
| | 15 | 00 | 28 | 00 | 35 | 12 | 01 | 20 | 32 |
| | 16 | 00 | 41 | 00 | 47 | 12 | 01 | 21 | 16 |
| | | | | | | | | 07 | 28 |
| | | | | | | | | 08 | 11 |
| | | | | | | | | 08 | 54 |

Figure 1601b. Extract from the Nautical Almanac daily pages for June 16, 2016.

column to estimate LAN unless extraordinary accuracy is required.

1602. Fundamental Systems of Time

Atomic based timekeeping is determined by the definition of the Systeme International (SI) second, with duration of 9,192,631,770 cycles of electromagnetic radiation corresponding to the transition between two hyperfine levels of the ground state of cesium 133. **International Atomic Time (TAI)** is an international time scale based on the non-stationary ensemble of atomic clock observations contributed by worldwide timekeeping laboratories, qualified by the Bureau International des Poids et Mesures (BIPM).

Universal time (UT) is a generic reference to one of several timescales that approximate the mean diurnal motion of the fictitious mean sun. Loosely, UT is **mean solar time at zero longitude**, or the **Greenwich meridian** (previously known as **Greenwich Mean Time (GMT)**). The term GMT has been dropped from scientific usage. In cur-

rent usage, UT either refers to **UT1** or **Coordinated Universal Time (UTC)**. In the navigational publications, UT always means UT1.

UT1 is a continuous timescale precisely defined by a mathematical expression that relates it to sidereal time, or the angle and rate of Earth's rotation to fixed points (usually very distant objects) of reference on the celestial background. Thus, UT1 is observationally determined by the apparent diurnal motions of celestial bodies and is affected by irregularities and the slowing of Earth's rate of rotation.

Coordinated Universal Time (UTC) is a discontinuous timescale determined by TAI and maintained by the BIPM. UTC is recognized by nearly all worldwide timing centers as the standard reference clock for purposes ranging from navigation to precise time stamping of financial transactions. UTC is accurately distributed (usually better than ± 1 ms) by radiometric and optical fiber-based transmission. UTC defines the 24 hour cycle or clock as 86,400 SI seconds, not related to the rotation rate of the Earth. In this way, UTC appears to run faster than UT1, although it is UT1 that is varying because of the slowing of the Earth. To maintain the long term coordination of UTC with UT1 to within ±0.9 seconds, a one second interval is typically added as necessary to UTC. This added interval is known as a leap second. Since the explicit synchronization of UTC and UT1 in 1958 through 2016, there have been 36 leap seconds inserted into UTC. Although the expectation of the leap second insertion should be regular, it is not, and it is this irregularity that makes the implementation of the leap second undesirable to the highly synchronized worldwide systems based on UTC. The leap second insertion is what characterizes UTC as a discontinuous time scale. The formal insertion of leap second is to expand the minute modulo by one (count the minute with a leap second as 58,59,60,00). Because the difference between UT1 and UTC are always less than 0.9 sec, navigators often do not need to account for the difference except when the highest precisions are required.

GPS Time is the time disseminated by the Navstar satellites of GPS, and is not UTC(USNO), meaning UTC as maintained by the United States Naval Observatory (USNO). Rather GPS Time is a continuous timescale monitored against the USNO master clock and maintained with a fixed offset of 19 seconds added to TAI. To formulate UTC, a leap second field is given within the navigation message of the GPS signal, which the receiver then uses to accordingly increment GPS Time. The need for a continuous timescale for Global Navigation Satellite Systems (GNSS), such as GPS Time, is necessary to allow for the determination of velocity and interaction with inertial navigation systems. In this way, real time system dynamics may be separated from the discrete time of day feature of GPS. See Section 1613 on dissemination systems for further details.

Terrestrial time (TT), formerly known as Terrestrial Dynamical Time (TDT), is rarely used by a navigator. In practice $TT = TAI + 32.184\ sec$.

Sidereal time is the hour angle of the vernal equinox. If the mean equinox is used (that is, neglecting nutation), the result is mean sidereal time; if the true equinox is used, the result is apparent sidereal time. The hour angle can be measured with respect to the local meridian or the Greenwich meridian, yielding, respectively, local or Greenwich (mean or apparent) sidereal times.

Delta T is the difference between Terrestrial Time and Universal Time: $\Delta T = TT - UT1$.

1603. Time and Longitude Arc

A navigator may be required to convert the measure of longitude arc to time or vice versa. The concept and math is not difficult, and calculators or tables (such as the one provided on page i in the back of the *Nautical Almanac*) can help. To illustrate, note that in this section, one day represents one complete rotation of the Earth as determined by a mean solar day. That is, one 24-hour period of 86,400 seconds is the same as the Earth rotating 360° . Therefore, the time of day is an indication of the phase (amount of rotation) within the Earth's orbital period, calculating how much of a mean solar day has elapsed, or what part of a rotation has been completed. For example, initialing the day at zero hours, at one hour later, the Earth has turned through $1/24$ of its rotation, or $1/24$ of 360° , or $360^\circ \div 24 = 15^\circ$

Smaller intervals can also be stated in angular units; since 1 hour or 60 minutes is equivalent to 15° of arc, 1 minute of time is equivalent to $15^\circ \div 60 = 0.25^\circ = 15'$ of arc, and 1 second of time is equivalent to $15' \div 60 = 0.25' = 15''$ of arc. Therefore any time interval can be expressed as an equivalent amount of rotation, and vice versa. Conversion among these units can be aided by the relationships indicated below, summarizing in table form:

| | | |
|-----------------|------------------|-------------------|
| 1 ^d | =24 ^h | =360 [°] |
| 60 ^m | =1 ^h | =15 [°] |
| 4 ^m | =1 [°] | =60' |
| 60 ^s | =1 ^m | =15' |
| 4 ^s | =1' | =60'' |
| 1 ^s | =15'' | =0.25' |

To convert time to arc:

If time is in hh:mm:ss format:

1. Convert to decimal hours. Take mm and divide by 60 (60 is the number of minutes per hour). Take ss and divide by 3600 (3600 is the number of seconds per hour). Add both to hh. Mathematically, decimal hours = $hh + mm \div 60 + ss \div 3600$.
2. Multiply decimal hours by 15 to obtain decimal degrees of arc.
3. If needed, convert decimal degrees of arc to $\text{deg}^\circ \text{amin}' \text{asec}''$ format, where deg is degree, amin is

minutes of arc, and asec is seconds of arc. To do this, deg is simply the integer portion of the decimal degrees. That is, the numbers before the decimal point. Take the remaining portion (that is, the decimal part) and multiply by 60. The minutes of arc, amin, is the integer portion of this. Take the remaining portion of this new value and again multiply it by 60. That is the seconds of arc, asec.

Example 1: Convert $14^h 21^m 39^s$ to arc.

Solution:

Step 1: Convert to decimal hours. $14 + 21 \div 60 + 39 \div 3600 = 14 + 0.35 + 0.01083 = 14.360833$ hours

Step 2: Multiply by 15. $14.360833 \times 15 = 215.4125^\circ$

Step 3: Convert to $\text{deg}^\circ \text{amin}' \text{asec}''$ format. The deg equal the integer portion of 215.4125, so $\text{deg} = 215$. The amin is found by taking the remainder, .4125, and multiplying it by 60; $.4125 \times 60 = 24.75$. The amin equals the integer part, so $\text{amin} = 24$. The asec is found by taking the remainder of that, .75, and multiplying it by 60, which equals 45, so $\text{asec} = 45$. The final answer is

$$14 \text{ h } 21 \text{ m } 39 \text{ s of time} = 215^\circ 24' 45''$$

To convert arc to time, the steps are similar.

If arc is in the $\text{deg}^\circ \text{amin}' \text{asec}''$ format:

Step 1: Convert to decimal degrees. To do this, take amin and divide by 60 (60 is the number of minutes of arc per degree). Take asec and divide by 3600 (3600 is the number of seconds of arc per degree). Add both to deg. Mathematically, decimal degrees = $\text{deg} + \text{amin} \div 60 + \text{asec} \div 3600$.

Step 2: Divide decimal degrees of arc by 15 to obtain decimal hours of time.

Step 3: If needed, convert decimal hours to hh:mm:ss format, where hh is hour, mm is minutes of time, and ss is seconds of time. To do this, hh is simply the integer portion of the decimal hours. That is, the numbers before the decimal point. Take the remaining portion (that is, the decimal part) and multiply by 60. The minutes of time, mm, is integer portion of this. Take the remaining portion of this new value and again multiply it by 60. That is the seconds of time, ss.

Convert $215^\circ 24' 45''$ to time units.

Step 1: Convert to decimal degrees. Decimal degrees = $\text{deg} + \text{amin} \div 60 + \text{asec} \div 3600$. In this example, $215 + (24 \div 60) + (45 \div 3600)$, which equals 215.4125 degrees.

Step 2: Divide decimal degrees of arc by 15 to obtain decimal hours of time. $215.4125 \div 15 = 14.360833$ hours.

Step 3: Convert decimal hours to hh:mm:ss format. The hh equal the integer portion of 14.360833, so $hh = 14$.

The mm is found by taking the remainder, .360833, and multiplying it by 60; $.360833 \times 60 = 21.65$. The mm equals the integer part, so mm = 21. The ss is found by taking the remainder of that, .65, and multiplying it by 60, which equals 39, so ss = 39. The final answer is

$$215^{\circ} 24' 45'' = 14 \text{ h } 21 \text{ m } 39 \text{ s}$$

Solutions can also be made using arc to time conversion tables in the almanacs. In the *Nautical Almanac*, the table given near the back of the volume is in two parts, permitting separate entries with degrees, minutes, and quarter minutes of arc. This table is arranged in this manner because the navigator converts arc to time more often than the reverse.

Convert $334^{\circ}18'22''$ to time units, using the *Nautical Almanac* arc to time conversion table.

Convert the 22" to the nearest quarter minute of arc for solution to the nearest second of time. Interpolate if more precise results are required.

$$334^{\circ} 00.00 \text{ m} = 22 \text{ h } 16 \text{ m } 00 \text{ s}$$

$$000^{\circ} 18.25 \text{ m} = 00 \text{ h } 01 \text{ m } 13 \text{ s}$$

$$334^{\circ} 18' 22'' = 22 \text{ h } 17 \text{ m } 13 \text{ s}$$

1604. Time Passage and Longitude

Section 1603 provides the direct coordination between the measure of longitude arc and the time passage of the mean solar day, equivalent to 24 hours equals 360 degrees of Earth's rotation. Thus, the measure of longitude between two fixed points is an angular equivalent of the time difference between these two points on the Earth. Therefore for any given time of day, places east of an observer have later time, and those places west have earlier time. The time difference observed between two places is equal to the difference of longitude between their meridians, expressed in units of time instead of arc. It is from this principle that longitude navigation through the use of a chronometer is derived. If an error free chronometer was set precisely at 12h at a given local noon, properly adjusted for the equation of time, then any longitudinal excursion (distance traveled east or west) could be determined through the interval of time passage on the chronometer, compared to the transit of the Sun across the new local (present) meridian.

1605. The Date Line

Since time accumulates later when traveling toward the east and earlier toward the west, a traveler circling the Earth gains or loses an entire day depending on the direction of travel. To provide a starting place for each new mean solar

day, a date line extending from Earth's poles is fixed by informal agreement, called the **International Date Line**. The International Date Line separates two consecutive calendar days. It coincides with the 180th meridian over most of its length. In crossing this line, the date is altered by one day. The date becomes one day earlier when traveling eastward from east longitude to west longitude. Conversely the date becomes one day later when traveling westward across it. When solving celestial problems, we convert local time to UTC and then convert this to local time on the opposite side of the date line.

1606. Civil Time vs. Mean Solar Time and Time Zones

Mean solar time is closely related to **civil time**, which is what our clocks read if they are set accurately. The worldwide system of civil time has historically been based on mean solar time, but in the modern system of timekeeping, there are some differences.

Civil time is based on a worldwide system of 1-hour time zone segments, which are spaced 15 degrees of longitude apart. (The time zone boundaries are usually irregular over land, and the system has broad variations; local time within a country is the prerogative of that country's government.) All places within a time zone, regardless of their longitudes, will have the same civil time, and when we travel over a time zone boundary, we encounter a 1-hour shift in civil time. The time zones are set up so that each is an integral number of hours from a time scale called Coordinated Universal Time (UTC). UTC is accurately distributed by GPS, the Internet, and radio time signals. So the minute and second "ticks" of civil time all over the world are synchronized and counted the same; it is only the hour count that is different. (There are a few odd time zones that are a $\frac{1}{4}$ or $\frac{1}{2}$ hour offset from neighboring zones. The minute count is obviously different in these places.)

1607. Zone Time

At sea, as well as ashore, watches and clocks are normally set to some form of **zone time (ZT)**. At sea the nearest meridian exactly divisible by 15° is usually designated as the **time meridian** or **zone meridian**. Thus, within a time zone extending $\pm 7.5^{\circ}$ on each side of the time meridian the time is the same, and time in consecutive zone increments differs by exactly one hour. The time maintained by a clock is changed as convenient, usually at a whole hour, when crossing the boundary between zones. Each time zone is identified by the number of times the longitude of its zone meridian is divisible by 15° , *positive in west longitude* and *negative in east longitude*. This number and its sign, called the **zone description (ZD)**, is the number of whole hours that are added to or subtracted from the zone time to obtain UTC. Note that the zone description does not change when Daylight Savings Time is in effect.

The mean fictitious sun is the celestial reference point for zone time. See Table 1607a and Figure 1607b for more detail.

| Time Zones, Zone Descriptions, and Suffixes | | | | | |
|---|-----|--------|----------------------|------|--------|
| ZONE | ZD | SUFFIX | ZONE | ZD | SUFFIX |
| 7.5° W to 7.5° E | 0 | Z | 7.5° W to 22.5° W | + 1 | N |
| 7.5° E to 22.5° E | -1 | A | 22.5° W to 37.5° W | + 2 | O |
| 22.5° E to 37.5° E | -2 | B | 37.5° W to 52.5° W | + 3 | P |
| 37.5° E to 52.5° E | -3 | C | 52.5° W to 67.5° W | + 4 | Q |
| 52.5° E to 67.5° E | -4 | D | 67.5° W to 82.5° W | + 5 | R |
| 67.5° E to 82.5° E | -5 | E | 82.5° W to 97.5° W | + 6 | S |
| 82.5° E to 97.5° E | -6 | F | 97.5° W to 112.5° W | + 7 | T |
| 97.5° E to 112.5° E | -7 | G | 112.5° W to 127.5° W | + 8 | U |
| 112.5° E to 127.5° E | -8 | H | 127.5° W to 142.5° W | + 9 | V |
| 127.5° E to 142.5° E | -9 | I | 142.5° W to 157.5° W | + 10 | W |
| 142.5° E to 157.5° E | -10 | K | 157.5° W to 172.5° W | + 11 | X |
| 157.5° E to 172.5° E | -11 | L | 172.5° W to 7.5° W | + 12 | Y |
| 172.5° E to 180° E | -12 | M | | | |

Note. - GMT is indicated by suffix Z. Standard times as kept in various places or countries are listed in *The Nautical Almanac* and *The Air Almanac*.

Table 1607a. Time zones, descriptions, and suffixes.

When converting ZT to UTC, a positive ZT is added and a negative one subtracted; converting UTC to ZT, a positive ZD is subtracted, and a negative one added.

Example: The UTC is 15^h27^m09^s.

Required: (1) ZT at long. 156°24.4' W

Solutions: 15 h 27 m 09 s - (+150/15) = 05 h 27 m 09 s

In example (1), the nearest 15° increment is 150°W, leaving a remainder of less than ±7.5° (+6.407°).

Example: The UTC is 15^h27^m09^s.

Required: (2) ZT at long. 039°04.8' E

Solutions: 15 h 27 m 09 s + (-45/15) = 18 h 27m 09 s

In example (2), the nearest 15° increment is 45°E, leaving a remainder of less than ±7.5° (-5.92°).

1608. Chronometer Time

Chronometer time (C) is time indicated by a chronome-

ter. Since a chronometer is set approximately to UTC and not reset until it is overhauled and cleaned about every 3 years, there is nearly always a **chronometer error (CE)**, either fast (F) or slow (S). The change in chronometer error in 24 hours is called **chronometer rate**, or daily rate, and designated gaining or losing. With a consistent error in chronometer rate of +1s per day for three years, the chronometer error would accumulate 18 minutes. Since chronometer error is subject to change, it should be determined from time to time, preferably daily at sea. Chronometer error can be determined by comparison to a radio derived time signal, by comparison with another timekeeping system of known error, or by applying chronometer rate to previous readings of the same instrument. It is recorded to the nearest whole or half second. Chronometer rate is recorded to the nearest 0.1 second/day.

Example: At UTC 1200 on May 12 the chronometer reads 12^h04^m21^s. At UTC 1600 on May 18 it reads 4^h04^m25^s.

Required:

1. Chronometer error at 1200 UTC May 12.
2. Chronometer error at 1600 UTC May 18.
3. Chronometer rate.
4. Chronometer error at UTC 0530, May 27.

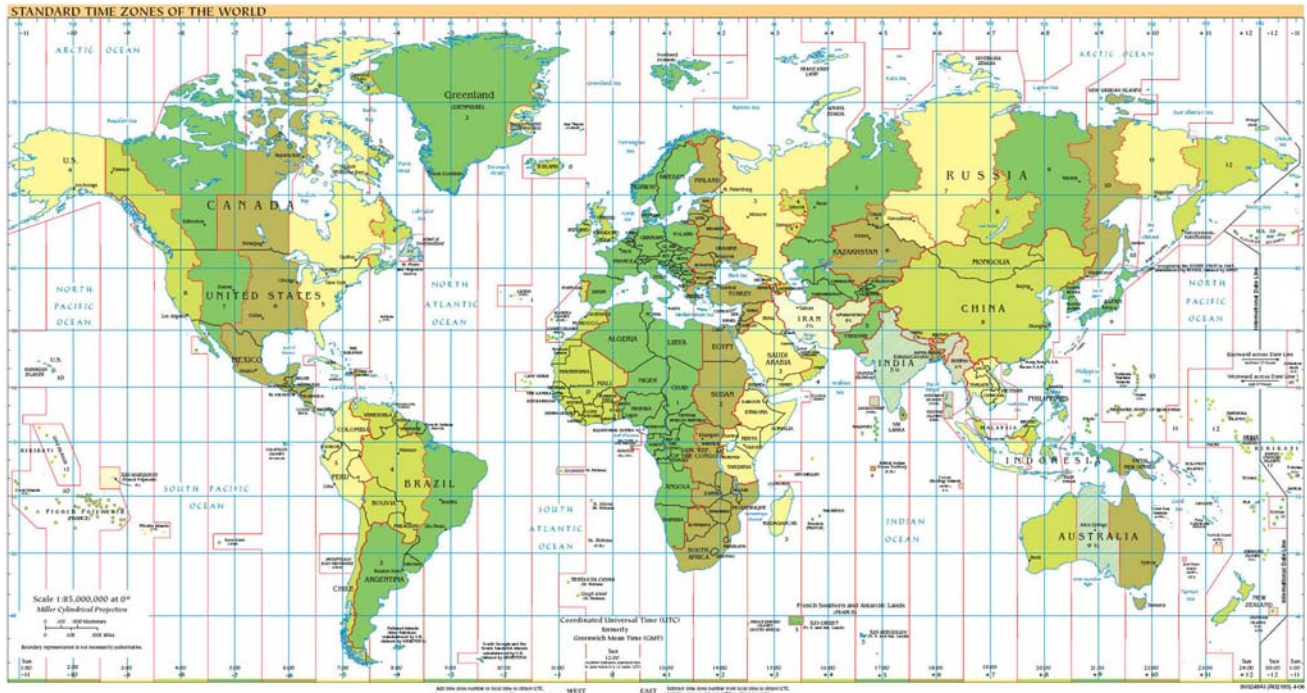


Figure 1607b. Time zones of the world.

Solutions:

1. UTC 12^h00^m00^s May 12
 C 12^h04^m21^s
 CE (F)4^m21^s gaining

2. UTC 16^h00^m00^s May 18
 C 04 04 25
 CE (F)4^m25^s gaining

3. UTC 18^d16^h present
 UTC 12^d12^h past
 diff. 06^d04^h = 6.2^d
 CE (F)4^m21^s 1200 May 12
 CE (F)4^m25^s 1600 May 18
 diff. 4^s (gained)
 daily rate 0.6^s/^d (gain) 4^s/6.2^d

4. UTC 27^d05^h30^m present
 UTC 18^d16^h00^m past
 diff. 08^d13^h30^m (8.5^d)
 CE (F)4^m25^s 1600 May 18
 corr. (+)0^m05^s diff. rate
 CE (F)4^m30^s 0530 May 27

Because UTC is on a 24-hour basis and chronometer time on a 12-hour basis, a 12-hour ambiguity exists. This is ignored in finding chronometer error. However, if chronometer error is applied to chronometer time to find UTC, a 12-hour error can result. This can be resolved by mentally applying the zone description to local time to obtain approximate UTC. A time diagram can be used for resolving doubt as to approximate UTC with date. If

the Sun for the kind of time used (mean or apparent) is between the lower branches of two time meridians (as the standard meridian for local time, and ZT 0 or Zulu meridian for UTC, the date at the place farther east is one day later than at the place farther west.

1609. Watch Time

Watch time (WT) is usually an approximation of zone time, except that for timing celestial observations it is easiest to set a comparing watch to UTC. If the watch has a second-setting hand, the watch can be set exactly to ZT or UTC, and the time is so designated. If the watch is not set exactly to one of these times, the difference is known as **watch error (WE)**, labeled fast (F) or slow (S) to indicate whether the watch is ahead of or behind the correct time.

If a watch is to be set exactly to ZT or UTC, set it to some whole minute slightly ahead of the correct time and stopped. When the set time arrives, start the watch and check it for accuracy.

The UTC may be in offset by 12^h, but if the watch is graduated to 12 hours, this will not be reflected. If a watch with a 24-hour dial is used, the actual UTC should be applied.

To determine WE, compare the reading of the watch with that of the chronometer at a selected moment. This may also be at some selected moment to UTC. Unless a watch is graduated to 24 hours, its time is designated as AM (ante meridian) before noon and PM (post meridian) after noon.

Even though a watch is set approximately to the zone

time, its error to UTC can be determined and used for timing observations. In this case the 12-hour ambiguity to UTC should be resolved, and a time diagram used to avoid miscalculation. This method requires additional work, and presents a greater probability of error, and gains no greater advantage provided through WE compensation.

If a stopwatch is used for timing observations, it should be started at some convenient UTC, such as a whole 5^m or 10^m. The time of each observation is then the UTC plus the watch time. Digital stopwatches and wristwatches are ideal for this purpose, as they can be set from a convenient UTC and read immediately after the observation is taken.

1610. Local Mean Time

Local mean time (LMT), like zone time, uses the mean Sun as the celestial reference point. It differs from zone time in that the local meridian is used as the terrestrial reference, rather than a zone meridian. Thus, the local mean time at each meridian differs from every other meridian, the difference being equal to the difference of longitude expressed in time units. At each zone meridian, including 0°, LMT and ZT are identical.

In navigation the principal use of LMT is in rising, setting, and twilight tables. The problem is usually one of converting the LMT taken from the table to ZT. At sea, the difference between the times is normally not more than 30^m, and the conversion is made directly, without finding GMT as an intermediate step. This is done by applying a correction equal to the difference of longitude. If the observer is west of the time meridian, the correction is added, and if east of it, the correction is subtracted. If Greenwich time is desired, it is found from ZT.

Where there is an irregular zone boundary, the longitude may differ by more than 7.5° (30^m) from the time meridian.

If LMT is to be corrected to daylight saving time, the difference in longitude between the local and time meridian can be used, or the ZT can first be found and then increased by one hour.

Conversion of ZT (including GMT) to LMT is the same as conversion in the opposite direction, except that the sign of difference of longitude is reversed. This problem is not normally encountered in navigation.

1611. Sidereal Time

Sidereal time uses the celestial datum of the vernal equinox (first point of Aries) as the celestial reference point instead of the apparent procession of Sun. Since the Earth revolves around the Sun, and since the direction of the Earth's rotation and revolution are the same, it completes a rotation with respect to the stars in less time (about 3 m 56.6 s of mean solar units) than with respect to the Sun, and during one revolution about the Sun (1 year) it makes one

complete rotation more with respect to the stars than with the Sun. This accounts for the daily shift of the stars nearly 1° westward each night. Hence, sidereal days are shorter than solar days, and its hours, minutes, and seconds are correspondingly shorter. Because of nutation, sidereal time is not quite constant in rate. Time based upon the average rate is called **mean sidereal time**, when it is to be distinguished from the slightly irregular sidereal time. The ratio of mean solar time units to mean sidereal time units is 1:1.00273791.

A navigator very seldom uses sidereal time.

1612. Time and Hour Angle

As mentioned earlier, **hour angle** is a measure of how far east or west of a meridian a celestial object appears. If the local meridian is used, this measure is called a **local hour angle (LHA)**. If the **Greenwich meridian** is used, then it is called a **Greenwich hour angle, GHA**. Hour angles are often expressed in arc units, between 0 and 360°. The hour angle is zero for an object crossing the meridian, and increases as the object moves west of the meridian (setting). In other words, an object transiting the meridian has an hour angle of 0°. Shortly after transit, its hour angle would be 1°, shortly before transit it would be 359°.

Sidereal time is the hour angle of the vernal equinox, but it is usually expressed in time units. Solar time at a specific location is also an hour angle measurement of the Sun, but since the day starts at midnight, 12 hours is added. That is, local solar time = 12 hours + local hour angle (expressed in hours) of the position of the Sun in the sky.

As with time, LHA at two places differs by their difference in longitude. In addition, it is often convenient to express hour angle in terms of the shorter arc between the local meridian and the body, that is, instead of 0° to 360°, it can be expressed 0° to 180°. This is similar to measurement of longitude from the Greenwich meridian. Local hour angle measured in this way is called meridian angle (*t*), which must be labeled east or west, like longitude, to indicate the direction of measurement. A westerly meridian angle is numerically equal to LHA, while an easterly meridian angle is equal to 360° - LHA. Mathematically, LHA = *t* (W), and LHA = 360° - *t* (E). Meridian angle is used in the solution of the navigational triangle.

Find LHA and t of the Sun at for long. 118°48.2' W if the GHA of the Sun is 231° 04.0'.

LHA = GHA - west longitude, and LHA = GHA + east longitude, thus, for our example

$$LHA(\text{Sun}) = 231^\circ 04.0' - 118^\circ 48.2' = 112^\circ 15.8'$$

$$t = 112^\circ 15.8' W$$

RADIO DISSEMINATION OF TIME SIGNALS

1613. Dissemination Systems

Of the many systems for time and frequency dissemination, the majority employ some type of radio transmission, either in dedicated time and frequency emissions or established systems such as radionavigation systems. The most accurate means of time and frequency dissemination today are by the mutual exchange of round-trip time signals through communication (commonly called Two-Way) and by the mutual observation of one-way signals from navigation satellites (such as Common View, All-in-View, and Differential GPS). One-way direct access to Global Navigation Satellite Systems (GNSS) is an excellent way to obtain UTC if many satellite observations are averaged.

Radio time signals can be used either to perform a clock's function or to set clocks. When using a radio wave several factors must be considered. One is the delay time of approximately 3 microseconds per kilometer it takes the radio wave to propagate and arrive at the reception point. Thus, a user 1,000 kilometers from a transmitter receives the time signal about 3 milliseconds later than the on-time transmitter signal. If time is needed to better than 3 milliseconds, a correction must be made for the time it takes the signal to pass through the receiver.

In most cases standard time and frequency emissions as received are more than adequate for ordinary needs. However, many systems exist for the more exacting scientific requirements, such as Precise Point Positioning using GNSS carrier phase.

1614. Characteristic Elements of Dissemination Systems

A number of common elements characterize most time and frequency dissemination systems. Among these elements, the most important are accuracy, ambiguity, repeatability or precision, coverage, availability of time signal, reliability, ease of use, cost to the user, and the number of users served. No single system optimizes all desired these characteristics. The relative importance of these characteristics will vary by application, and the solution for one user may not be satisfactory to another. These trade among these common elements are discussed in the following examination of a hypothetical radio signal.

Consider a very simple system consisting of an unmodulated 10-kHz signal as shown in Figure 1614. This signal, leaving the transmitter at 0000 UTC, will reach the receiver at a later time due to the propagation delay. The user must know this delay because the accuracy of the recovered time from the transmitted signal can be no better than the certainty in this delay. Since all cycles of the signal waveform are identical, the signal is ambiguous and

the user must resolve which cycle is the "on time" cycle, in this case the cycle leaving at 0000 UTC. This means, with respect to a 10-kHz signal waveform, the user must already know the propagation delay to within ± 50 microseconds (half the period of the signal). The calibration of the waveform cycle over cycle phase (zero crossings as defined in the figure) to resolve ambiguity in time dissemination is called the "tick to phase" determination. Further, the user may desire to periodically use the time-transfer system, say once a day, to check their clock or frequency standard. However, if the travel delay and instrument repeatability vary from one day to the next without the user knowing or correcting, the accuracy will be limited by the amounts attributed to these uncertainties.

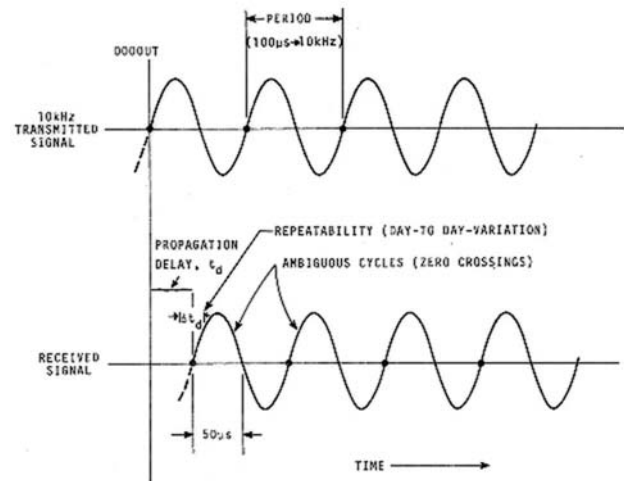


Figure 1614. Single tone time dissemination.

Many users are interested in making time-coordinated measurements over large geographic areas. They would like all measurements to be traceable to one master reference time to make corrections for the offsets between geographically distributed timekeeping systems. In addition, traceability to a master reference system increases confidence that all time measurements are related to each other in a consistent manner. Thus, the accuracy over the range of geographic coverage of a dissemination system is an important characteristic. Another important characteristic of a time dissemination system is the percentage of availability.

For most social uses of time, people who have to keep an appointment usually need to know the time of day to within a few minutes. Although requiring only coarse time information, people keeping a social schedule want it on demand, and thus carry a wristwatch or other portable device with a clock function that gives the time with continuous availability. People who have access to the internet can set the time of their personal computers to an

accuracy of UTC of ± 100 milliseconds, or considerably better, through the Network Time Protocol (NTP), with near continuous availability, dependent on the network's reliability. On the other hand, a person with a scientific interest may possess a very good clock capable of maintaining a few microseconds with only an occasional need for an accuracy update, perhaps only once or twice a day. However, in this distinguishing case, the scientific user requires much greater precision and accuracy in time dissemination than the social user, when available. This leads to the characteristic of time dissemination reliability, i.e., the likelihood that a time signal will be available when scheduled. In the case of the scientific user, the availability of time dissemination may be a critical operational need, and reliability may be as important as precision. Propagation fade-out or user location (such as in a basement or the woods) can sometimes prevent or distort signal reception. Thus, the quality and cost of time dissemination services contrast accuracy, availability and reliability against the application needs of the user community and the capability of their local clocks.

1615. Radio Wave Propagation Factors

Radio has been used to transmit standard time and frequency signals since the early 1900's. As opposed to the physical transfer of time via portable clocks, the transfer of timing information by radio involves the use of electromagnetic propagation from a transmitter, usually carrying the master reference time, to a navigator's receiver at long distance.

In the broadcast of frequency and time over radio, the transmitted signals are directly related to some master clock and are usually received with some degradation in accuracy. In a vacuum and with a noise-free background, timing signals should be received at a distant receiver essentially undistorted, with a constant path delay due to the propagation of the radio wave at the speed of light (299,773 kilometers per second). However the propagation media, including the Earth's atmosphere and ionosphere, reflections and refractions caused by man-made obstructions and geographic features, and space weather (solar-activity), as well as the inherent characteristics of transmitters and receivers, degrade the fidelity and accuracy of timing derived from the received radio signals. The amount of degradation in timing recovered from the signals is also dependent upon the frequency of the transmitted radio wave (carrier frequency), and the length of signal path. In many cases the application of propagation models or supplementary information must be used to correct for the distorting effects. For example, GPS receivers, which only use the L1 frequency, have correction models built into their systems to correct for propagation through the ionosphere from space

Radio dissemination systems can be classified in a number of different ways. One way is to separate those carrier frequencies low enough to be reflected by the

ionosphere (below 30 MHz) from those sufficiently high to penetrate the ionosphere (above 30 MHz). The former can be observed at great distances from the transmitter but suffer from ionospheric propagation distortion that limits accuracy; the latter are restricted to line-of-sight applications but show less signal degradation caused by propagation effects. The most accurate systems tend to be those which use the higher, line-of-sight frequencies, and with the advent of space-based satellite navigation, such as GPS, these also have promoted the most users and applications for radio time dissemination.

1616. Standard Time Broadcasts

The World Radiocommunication Conference (WRC), is the means by which the International Telecommunications Union (ITU), allocates certain frequencies in five bands for standard frequency and time signal emission. For such dedicated standard frequency transmissions, the ITU Radiocommunication Sector (ITU-R) recommends that carrier frequencies be maintained so that the average daily fractional frequency deviations from the internationally designated standard for measurement of time interval should not exceed \pm ten parts per trillion.

1617. Time Signals

The modern method of determining chronometer error and daily rate is by comparison to time recovered from radionavigation signals. The most accurate and readily available method for vessels is from navigation receivers of GPS, or other GNSS, and/or, where available, Enhanced Long Range Navigation (eLORAN) signals. Also, many maritime nations broadcast time signals on short-wave frequencies, such as the U.S. station (WWV), or German station (DCF77). Further discussion can be found in *NGA Pub. 117, Radio Navigational Aids* and the British Admiralty *List of Radio Signals*. A list of signals transmitted by timing labs is published in the Annual Report of the International Bureau of Weights and Measures (BIPM). The BIPM report is currently available on the Internet (see Figure 1617a). An important reason for employing more than one technique is to guard against both malfunction in equipment or malicious interference, such as spoofing.



Figure 1617a. BIPM Annual Report on Time Activities.
<http://www.bipm.org/en/bipm/tai/annual-report.html>

If a vessel employs a mechanically actuated (main-spring) chronometer or even an atomic clock, the time

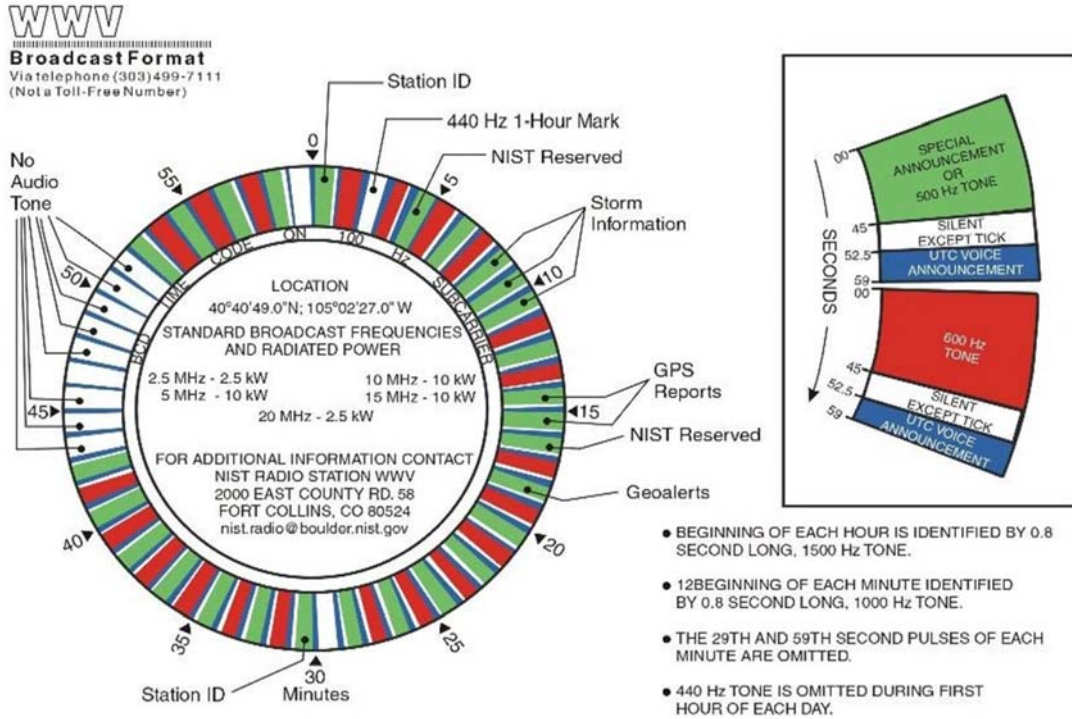


Figure 1617b. Broadcast format of station WWV.

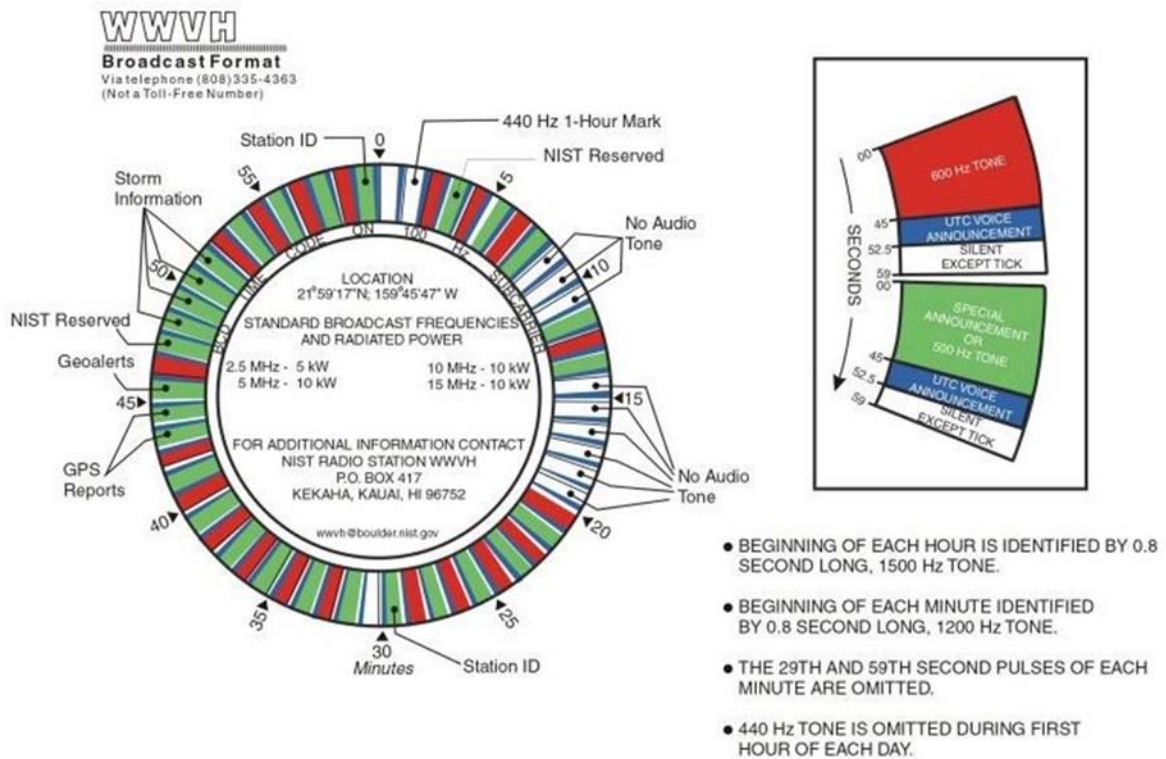


Figure 1617c. Broadcast format of station WWVH.

should nonetheless be checked daily against a time derived from radio signals, beginning at least three days prior to departure. The offset and computed rate should be entered in the chronometer record book (or record sheet) each time they are determined, although for an atomic clock the main concern is catastrophic or end of life failure. For example, cesium-beam tube atomic clocks have a limited life due to the consumption of the cesium metal during extended operation, typically 5 to 7 years.

For the U.S. the National Institute of Standards and Technology (NIST) broadcasts continuous time and frequency reference signals from WWV, WWVH, and WWVB. Because of their wide coverage and relative simplicity, the HF services from WWV and WWVH are used extensively for navigation. Station WWV broadcasts from Fort Collins, Colorado at the internationally allocated frequencies of 2.5, 5.0, 10.0, 15.0, and 20.0 MHz; station WWVH transmits from Kauai, Hawaii on the same frequencies with the exception of 20.0 MHz. The broadcast signals include standard time and frequencies, and various voice announcements. Details of these broadcasts are given in NIST Special Publication 432, NIST Frequency and Time Dissemination Services. Both HF emissions are derived from cesium beam atomic frequency standards with traceable reference to the NIST atomic frequency and time standards.

The time ticks in the WWV and WWVH emissions are shown in Figure 1617b and Figure 1617c. The 1-second UTC markers are transmitted continuously by WWV and WWVH, except for omission of the 29th and 59th marker each minute. With the exception of the beginning tone at each minute (800 milliseconds) all 1-second markers are of 5 milliseconds duration and at a tone of 440 Hz. Each pulse is preceded by 10 milliseconds of silence and followed by 25 milliseconds of silence. Time voice announcements are given also at one minute intervals. All time announcements are UTC.

As explained in the next section, Coordinated Universal Time (UTC) may differ from (UT1) by as much as 0.9 second; the actual difference can be found at IERS web pages Bulletin A, which published on the internet at

<http://datacenter.iers.org/eop/-/somos/5Rgv/latest/6>. NGA Pub. No. 117, *Radio Navigational Aids*, should be referred to for further information on time signals.

1618. Leap-Second Adjustments

By international agreement, UTC is maintained to be no more than ± 0.9 seconds from agreement with the continuous celestial timescale, UT1. The introduction of leap seconds allows a clock maintaining UTC to stay approximately coordinated with mean solar time or stay near the procession of the fictitious mean sun across the sky. The insertion of leap seconds makes UTC a discontinuous timescale. The main contributor to the need for a leap second adjustment is the slowing of the Earth's rotation at about 1.7 ms/century. However, because of irregular variations in the yearly rate of the rotation of the Earth, year over year occurrences of the insertion of a leap seconds is not predictable.

The Central Bureau of the International Earth Rotation and Reference Frames Service (IERS) decides upon and announces the introduction of a leap second. The IERS announces the leap second insertion at least eight weeks in advance. Because of the irregularity of the Earth's rotation, the IERS provides that a second may be advanced or retarded, positive or negative leap second, though a negative leap second has never been required since its institution in 1972. The leap second is introduced as the last second of a UTC month, but first preference is given to the end of December and June, and second preference is given to the end of March and September. A positive leap second begins at 23 h 59 m 60 s and ends at 00 h 00 m 00 s of the first day of the following month. In the case of a negative leap second, 23 h 59 m 58 s is followed one second later by 00 h 00 m 00 s (skipping 23 h 59 m 59 s) of the first day of the following month. Leap second adjustments of UTC are performed uniformly, and in synchrony (per interval of a SI second) across the world.

The dating of events in the vicinity of a leap second is effected in the manner indicated in Figure 1618a and Figure 1618b.

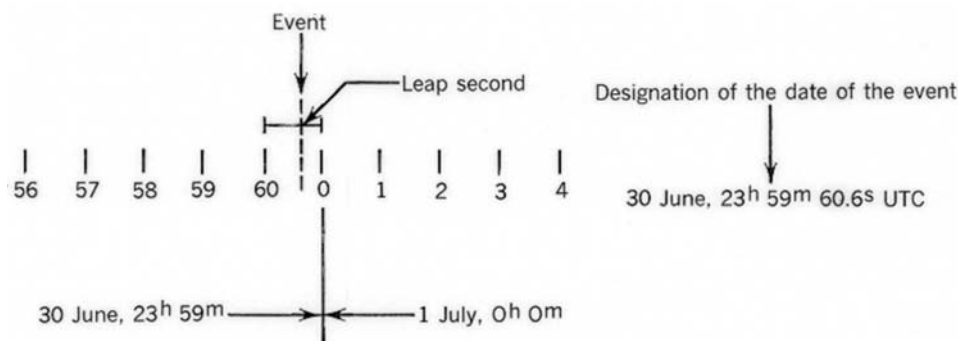


Figure 1618a. Dating of event in the vicinity of a positive leap second.

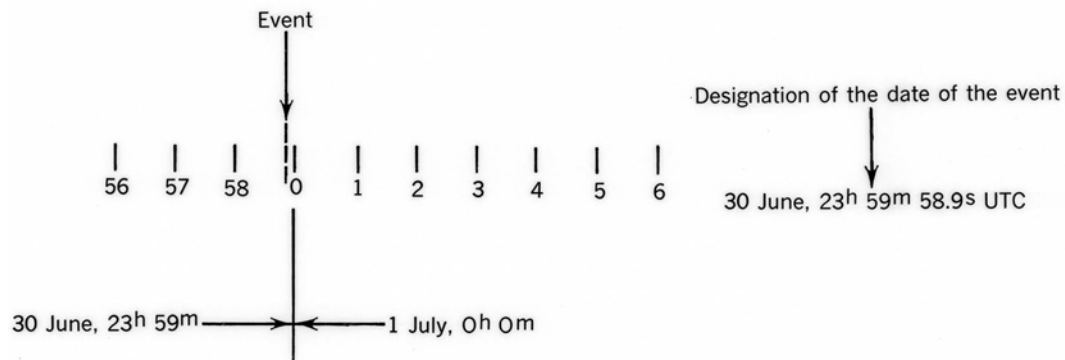


Figure 1618b. Dating of event in the vicinity of a negative leap second.

Whenever a leap second adjustment is to be made to UTC, navigators are advised by information presented on the web pages of the United States Naval Observatory, USNO, IERS Bulletin C and the International Bureau of Weights and Measures (BIPM). Additional information is available on the USNO and IERS webpages (see Figure 1618c and Figure 1618d).



Figure 1618c. USNO leap second data between January 1972 and January 2017. Link: <http://tycho.usno.navy.mil/leapsec.html>

1619. Use of Time, Time-interval, and other Novel Techniques for Approximate Determination of Chronometer Time, Latitude, and Longitude.

There may arise situations in which a mariner needs to address the problem of determining date, time, latitude, and longitude using only minimal resources and with little, if any, prior knowledge of the values of these parameters. Given this, it is useful to consider the value of using simple time, “time-interval”, azimuth, “azimuth interval”, and “instrument-free” or “instrument-limited” measurements, performed in conjunction with table look-up of data from the Air or Nautical Almanacs and/or back-of-the-envelope computations. The term “instrument-limited”, in this context, applies when azimuth readings are made with a simple compass, and elevation readings are accomplished using a handheld inclinometer rather than a sextant or tripod-mounted surveying transit.

Figure 1619 illustrates a convenient instrument, which is a combined inclinometer/compass that can be used on land without a clearly defined horizon, and at night using

internal illumination. One of the user's eyes reads the internal scales while the other eye lines the internal graticule up with the star or other object being measured. The human ability to merge the different optical images into one perceived image is not universal. Up to 15% of individuals are unable to merge the different visual images. Although not of sextant accuracy, the device is rugged and portable, and is precise to about 1 degree for handheld use without a tripod.



Figure 1619. Combined compass/inclinometer with internal lighting and automatic leveling.

Note that a modern smart phone, with its built in clock, camera, inclinometer, and compass can be used for the same purpose if GPS is denied, and can also be programmed with a star atlas, almanac data, and navigation algorithms. However, the successful use of a smartphone as a combined sextant, chronometer, and navigation computer depends critically on battery life.

The level of precision of an inclinometer and compass

can be compared with the celestial measurements described previously as follows. Sextant measurements typically have a best-case precision of 0.2 minutes of arc. Related time measurements are typically accomplished with a resolution of one second. Note that 1 minute of arc at the equator corresponds to a distance of one nautical mile and equates to four seconds of clock time. Thus, it takes 4 seconds for the earth to rotate one arc-second around its axis.

With respect to precision measurement of time, knowledge of Greenwich or Universal time is typically specified to less than quartz clock accuracy (i.e., to about one second resolution). When there is a clock offset bias, its value and drift rate are typically known. It will often be the case that local time is synchronized to Greenwich time within one second, even for everyday consumer applications, and far better than this for time signals disseminated from a wireless network to one's cell phone.

Note that the poor relative precision of a magnetic compass with respect to that of a sextant precludes the combined use of sextant and azimuth measurements. However, when an inclinometer of limited precision is the best available instrument, it can also be beneficial to include compass-derived azimuth measurements of comparable precision.

With this background, some useful examples of relatively simple, but in certain situations of great value, navigation techniques are presented.

1. **Quick observation of Polaris and the northern sky to approximate latitude and longitude.** To estimate latitude simply make an observation (when in the northern hemisphere) of Polaris, the north star. If Greenwich or universal time is available using a simple quartz watch or cell phone, longitude can be inferred. This can be done with the help of a star chart, by observing the “clock angles” of the constellation Cassiopeia and Canis Major (the big dipper). Experienced viewers of the night sky routinely estimate time by unassisted observations of the moon and of the constellations of the Zodiac.
2. **Noon observation of the sun to compare with an observation of Polaris to determine solar declination, and hence to determine approximate date and time.** During daylight hours, the maximum angle of the sun above the horizon at *local apparent noon* can be determined by a series of measurements made at time intervals of a few minutes. The highest elevation angle of the sun, $Elevation_{sun}$, occurs at local noon when the sun is due south of the observer. This measurement, combined with the estimate of latitude from measurement of the north star, Polaris, yields the declination of the sun. Specifically, the latitude value obtained from measurement of Polaris is

related to the solar declination by the equation:

$$90 \text{ degrees} - Elevation_{sun} + declination = Latitude$$

The declination depends on time, but not on the observer's position. An approximate measurement of the declination can be matched to the daily tables in the Nautical Almanac to yield the date, and within a few hours, a value for Universal Time (which in this context can be regarded as being equivalent to Greenwich Mean Time, or GMT). For example, the elevation of the sun on September 30, 2016 measured at 1700 hours GMT is computed, from the Nautical Almanac, to be 47:50:30 deg:min:sec with an azimuth of 180.8 degrees, indicating that the measurement is made at a time that is very close to local apparent noon. Using the equation above, we deduce that the declination is $latitude + elevation - 90 = 39:00:00 + 47:50:30 - 90:00:00 = -3:09:30$, in very close agreement with the Nautical Almanac lookup value of -3:09:18 deg:min:sec.

Using this value of declination to identify a table entry in the Nautical Almanac takes one immediately to the daily entry for September 30, 2016 at 1700 hours universal time (e.g., GMT), thus illustrating the causal relationship between solar declination and date and time. Once GMT is known, the traditional methods of determining latitude and longitude using the stars, planets, and/or sun can be implemented.

Of course, if one knows Greenwich time to high precision from, for example, a digital watch, this same measurement, in conjunction with another measurement of the sun at a different point in time, yields the traditional *running fix*, which lies in the purview of the earlier sections of this chapter.

3. **Observations of sunrise and sunset to determine longitude.** If Greenwich time is known from a digital watch and an intelligent estimate of the relevant time zone, a simpler implementation of the running fix is easily accomplished. In this case, one measures only the times of sunrise and sunset, neither of which requires a sextant or artificial horizon when a clear horizon is available (i.e., on or near the ocean or other large body of water). The value for local noon is given as the midpoint in local time of the sunrise and sunset measurements. When this value is corrected to Greenwich time by the appropriate time zone corrections, the longitude is estimated by multiplying the time of local noon by 15 degrees per hour.

A better estimate of longitude is then obtained by adding/subtracting the requisite correction for the *equation of time*. This is found on the daily page of the Nautical Almanac for the date and time of the observation, and is added/subtracted to the value of

time that is then multiplied by the factor of 15 degrees per hour.

For example, the Washington Post newspaper provides daily values for local sunrise and sunset. On September 30, 2016, these are given as 7:03 A.M. and 6:52 P.M. EDT. Subtracting 1 hour to change to standard time, then taking the midpoint time yields a value for local noon of 11:57:30 h:m:s. Adding 10 minutes as the approximate correction for the equation of time (taken from the Nautical Almanac daily page for September 30th) corrects the time of local noon to GMT/UT, resulting in a value of 12:07:30 h:m:s.

If Washington was precisely 5 times zones away from Greenwich, then local noon in Washington would occur at 12:00:00 local time, after correcting for the equation of time. Five time zones, at 15 degrees per hour, is 75 degrees of longitude. Adding the additional 7.5 minutes corresponds to an additional 1.9 degrees of longitude, yielding a putative value for the longitude of Washington D.C. of 77 degrees West. (Note that the Naval Observatory, USNO, is at 38.9217° N, 77.0669° W).

4. **Compass measurement of the azimuth to Polaris to determine latitude and magnetic variation in order to determine position when latitude is already known.** Measurements made using a simple compass can be surprisingly useful. Measurement of the bearing of Polaris can be used to determine the local value of magnetic variation. Combined with an observation of latitude using an inclinometer or sextant, a map of magnetic variation versus latitude can then be used to generate an approximate position measurement.

In cases where magnetic variation is not known, relative bearing measurements yield “azimuth interval” measurements which remove the common mode error due to magnetic variation. In any case, the approach described herein is used routinely for pointing certain types of portable satellite telephone terminals at the appropriate satellite location in the geostationary arc.

5. **“Guess and Test” using simple Nautical Almanac equations in order to take advantage of combined elevation and azimuth measurements.** *Nautical Almanac* computations can be quite complicated. For the purposes of this section, a

convenient path forward is to use the straightforward equations for computing the calculated values of elevation angle H_c and azimuth Z from assumed values of the time, Greenwich Hour Angle (GHA), Sidereal Hour Angle (SHA), and declination for the celestial objects of interest. The relevant equations are given in the *Nautical Almanac* and are readily implemented using a calculator or perhaps a smartphone “App”.

Rather than use traditional iterative computations, this approach requires one to *guess* an “assumed position” and *test* the computed values of elevation H_c and azimuth Z against their measured values. One utility is that this provides a convenient way to integrate compass measurements of azimuth, corrected for magnetic variation as described above, into the data stream. The benefit is that a single sighting of the sun, if an azimuth measurement is included, provides the two data points needed to compute a latitude and longitude fix. There simply may not be time, or suitable weather conditions, to compute a running fix. The running fix, as described above, requires multiple measurements of the sun at widely spaced intervals of time.

6. **Cloudy night celestial navigation.** On a cloudy night, when only a single star is visible through a break in the clouds, a single measurement of the elevation and azimuth to a star lets one compute a location fix. Even if the identity of the star is not known, it is possible to perform the H_c and Z computations, for the assumed position, for several stars. Then the star whose measurement yields the most plausible position fix can often be reliably be assumed to be the star that was actually observed. Note that even a poor measurement of azimuth can be used to help identify the name, and hence the correct declination and sidereal hour angle values, to be used in the position computation.

There are many variations and extensions of these techniques and methods. The combination of a precision time reference and an accurate sextant is regaining favor after decades of single-system dependence on GPS, and more recently, E-LO-RAN. In extremis, and with little practice, even a combination of a protractor with a home-made plumb bob and a simple pendulum of length L and period $2\pi\sqrt{L/g}$ might bring one safely home.