

CHAPTER 15

AZIMUTHS AND AMPLITUDES

INTRODUCTION

1500. Checking Compass Error

The navigator must constantly be concerned about the accuracy of the ship's primary and backup compasses, and should check them regularly. A regularly annotated compass log book will allow the navigator to notice a developing error before it becomes a serious problem.

As long as at least two different types of compass (e.g. mechanical gyro and flux gate, or magnetic and ring laser gyro) are consistent with each other, one can be reasonably sure that there is no appreciable error in either system. Since different types of compasses depend on different scientific principles and are not subject to the same error sources, their agreement indicates almost certainly that no error is present.

A navigational compass can be checked against the heading reference of an inertial navigation system if one is installed. One can also refer to the ship's indicated GPS track as long as current and leeway are not factors, so that the ship's COG and heading are in close agreement.

The navigator's only completely independent directional reference (because it is extra-terrestrial and not man-made) is the sky. The primary compass should be

checked occasionally by comparing the observed and calculated azimuths and amplitudes of a celestial body. The difference between the observed and calculated values is the compass error. This chapter discusses these procedures.

Theoretically, these procedures work with any celestial body. However, the Sun and Polaris are used most often when measuring azimuths, and the rising or setting Sun when measuring amplitudes.

While errors can be computed to the nearest tenth of a degree or so, it is seldom possible to steer a ship that accurately, especially when a sea is running, and it is reasonable to round calculations to the nearest half or perhaps whole degree for most purposes.

Various hand-held calculators and computer programs are available to relieve the tedium and errors of tabular and mathematical methods of calculating azimuths and amplitudes. Naval and Coast Guard navigators will find the STELLA program useful in this regard. This program is discussed in further in [Chapter 19 - Sight Reductions](#). STELLA stands for the System to Establish Latitude and Longitude Astronomically.

AZIMUTHS

1501. Gyro Error by Azimuth of the Sun

Mariners not having STELLA may use *Pub 229, Sight Reduction Tables for Marine Navigation* to compute the Sun's azimuth. They compare the computed azimuth to the azimuth measured with the compass to determine compass error. In computing an azimuth, interpolate the tabular azimuth angle for the difference between the table arguments and the actual values of declination, latitude, and local hour angle. Do this triple interpolation of the azimuth angle as follows:

1. Enter the *Sight Reduction Tables* with the integer values of declination, latitude, and local hour angle. For each of these arguments, extract a base azimuth angle.
2. Reenter the tables with the same latitude and LHA arguments but with the **declination** argument 1° greater than the base declination argument. Record the difference between the respondent azimuth

angle and the base azimuth angle and label it as the azimuth angle difference (Z Diff.).

3. Reenter the tables with the base declination and LHA arguments, but with the **latitude** argument 1° greater than the base latitude argument. Record the Z Diff. for the increment of latitude.
4. Reenter the tables with the base declination and latitude arguments, but with the LHA argument 1° greater than the base LHA argument. Record the Z Diff. for the increment of LHA.
5. Determine the increment of each of the three arguments, which is simply the actual minus the base argument. Use this to compute a correction for each of the three arguments, using the formula $\text{Correction} = \text{Z Diff.} \times (\text{increment} \div 60)$.
6. Sum the three corrections to obtain a total correction. Apply this to the base azimuth angle to obtain the true azimuth angle.

The gyrocompass error is the difference between the true azimuth angle (Zn) and the measured azimuth angle per gyrocompass (Zn pgc). The gyro error is West if the gyro reads a larger value than true; it is East if the gyro reading is the smaller value. One way to remember this is “gyro

best, error West; gyro least, error East.” (Be careful of this mnemonic near the 360°/0° boundary, though, where a value slightly larger than 0° is actually treated as slightly larger than 360°.)

| | Actual | Base Arguments | Base Z | Tab* Z | Z Diff. | Increments | Correction Z Diff x (Inc. 60) |
|------------|------------|----------------|--------|--------|---------|-------------|-------------------------------|
| Dec. | 8°47.4' S | 8° | 121.3° | 122.3° | +1.0° | 47.4' | +0.8° |
| DR Lat. | 23°55.0' N | 23°(Contrary) | 1213° | 122.0° | +0.7° | 55.0' | +0.6° |
| LHA | 317°37.4' | 317° | 1213° | 122.1° | +0.8° | 37.4' | +0.5° |
| | | | | | | Total Corr. | +1.9° |
| Base Z | 121.3° | | | | | | |
| Corr. | (+) 1.9° | | | | | | |
| Z | N 121.3°E | | | | | | |
| Zn | 123.2° | | | | | | |
| Zn pgc | 124.0° | | | | | | |
| Gyro Error | 0.8° W | | | | | | |

*Respondent for the two base arguments and 1° change from third base argument, in vertical order of Dec., DR Lat., and LHA.

Figure 1501. Azimuth by Pub. No. 229.

Example:

In DR latitude 23° 55.0'N, the azimuth of the Sun is 124.0° per gyrocompass (pgc). At the time of the observation, the declination of the Sun is 8° 47.4'S; the local hour angle of the Sun is 317° 37.4. Determine gyro error.

Solution:

(See Figure 1501) Enter the actual value of declination, DR latitude, and LHA. Round each argument down to the nearest whole degree. Enter the Sight Reduction Tables with these whole degree arguments and extract the base azimuth value for these rounded off arguments. Record the base azimuth value in the table.

As the first step in the triple interpolation process, increase the value of the declination by 1° to 9° because the actual declination was greater than the base declination. Enter the Sight Reduction Tables with the following arguments: (1) Declination = 9°; (2) DR Latitude = 23°; (3) LHA = 317°. Record the tabulated azimuth for these arguments.

As the second step in the triple interpolation process, increase the value of latitude by 1° to 24° because the actual DR latitude was greater than the base latitude. Enter the Sight Reduction Tables with the following arguments: (1) Declination = 8°; (2) DR Latitude = 24°; (3) LHA =

317°. Record the tabulated azimuth for these arguments.

As the third and final step in the triple interpolation process, increase the value of LHA to 318° because the actual LHA value was greater than the base LHA. Enter the Sight Reduction Tables with the following arguments: (1) Declination = 8°; (2) DR Latitude = 23°; (3) LHA = 318°. Record the tabulated azimuth for these arguments.

Calculate the Z Difference by subtracting the base azimuth from the tabulated azimuth. Be careful to carry the correct sign.

$$Z \text{ Difference} = \text{Tab Z} - \text{Base Z}$$

Next, determine the increment for each argument by taking the difference between the actual values of each argument and the base argument. Calculate the correction for each of the three argument interpolations by dividing the increment by 60, and multiplying the result by the Z difference.

The sign of each correction is the same as the sign of the corresponding Z difference used to calculate it. In the above example, the total correction sums to +1.9. Apply this value to the base azimuth of 121.3° to obtain the true azimuth 123.2°. Compare this to the gyrocompass reading of 124.0° pgc. The compass error is 0.8°W.

AZIMUTH OF POLARIS

1502. Gyro Error by Azimuth of Polaris

The Polaris tables in the *Nautical Almanac* list the azimuth of Polaris for latitudes between the equator and 65° N. Figure 1912b in Chapter 19 shows this table. Compare a compass bearing of Polaris to the tabular value of Polaris to determine compass error. The entering arguments for the table are LHA of Aries and observer latitude.

Example:

On February 23, 2016, at LAT 29°31.0' N and LON 074°30.0'W, at 04-21-15 GMT, Polaris bears 359.9° pgc. Calculate the gyro error.

Date 23 February 2016
Time (GMT) 04-21-15

GHA Aries 217° 49.3' (from almanac daily pages & interpolation)
 Longitude 074° 30.0'W
 LHA Aries 143° 19.3'

down and extract the value for the given latitude. Since the increment between tabulated values is so small, visual interpolation is sufficient. In this case, the azimuth for Polaris for the given LHA of Aries and the given latitude is 359.2°.

Solution:

Enter the azimuth section of the Polaris table with the calculated LHA of Aries. In this case, go to the column for LHA Aries between 140° and 149°. Follow that column

Tabulated Azimuth 359.2°T
 Gyrocompass Bearing 359.9°T
 Error 0.7°W

AMPLITUDES

1503. Amplitudes

A celestial body's amplitude angle is the complement of its azimuth angle. At the moment that a body rises or sets, the amplitude angle is the arc of the horizon between the

body and the East/West point of the horizon where the observer's prime vertical intersects the horizon (at 90°), which is also the point where the plane of the equator intersects the horizon (at an angle numerically equal to the observer's co-latitude). See Figure 1503.

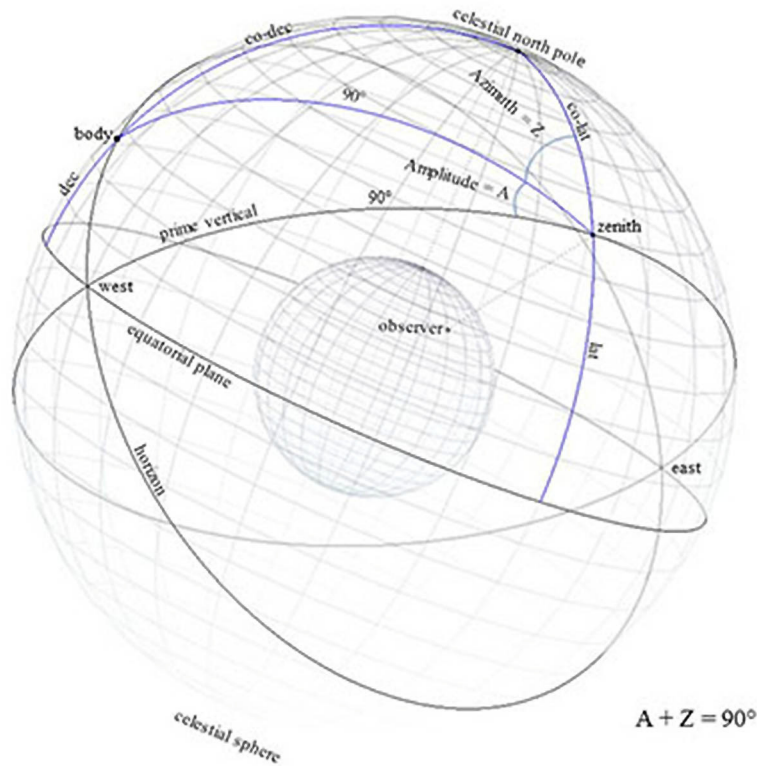


Figure 1503. The amplitude angle (A) subtends the arc of the horizon between the body and the point where the prime vertical and the equator intersect the horizon. Note that it is the complement of the azimuth angle (Z).

In practical navigation, a bearing (psc or pgc) of a body can be observed when it is on either the celestial or the visible horizon. To determine compass error, simply convert the computed amplitude angle to true degrees and compare it with the observed compass bearing.

The angle is computed by the formula:

$$\sin A = \sin Dec / \cos Lat.$$

This formula gives the angle at the instant the body is on the celestial horizon. It does not contain an altitude term because the body's computed altitude is zero at this instant.

The angle is prefixed E if the body is rising and W if it is setting. This is the only angle in celestial navigation referenced FROM East or West, i.e. from the prime vertical. A body with northerly declination will rise and set

North of the prime vertical. Likewise, a body with southerly declination will rise and set South of the prime vertical. Therefore, the angle is suffixed N or S to agree with the name of the body's declination. A body whose declination is zero rises and sets exactly on the prime vertical.

Due largely to refraction, dip, and its disk size, the Sun is on the celestial horizon when its lower limb is approximately two thirds of a diameter above the visible horizon. The Moon is on the celestial horizon when its upper limb is on the visible horizon. Stars and planets are on the celestial horizon when they are approximately one Sun diameter above the visible horizon.

When observing a body on the visible horizon, a correction from [Table 23 - Correction of Amplitude as Observed on the Visible Horizon](#) must be applied. This correction accounts for the slight change in bearing as the body moves between the visible and celestial horizons. It reduces the bearing on the visible horizon to the celestial horizon, from which the table is computed.

For the Sun, stars, and planets, apply this correction to the observed bearing in the direction away from the elevated pole. For the moon, apply one half of the correction toward the elevated pole. Note that the algebraic sign of the correction does not depend upon the body's declination, but only on the observer's latitude. Assuming the body is the Sun the rule for applying the correction can be outlined as follows:

| Observer's Lat. | Rising/Setting | How to Apply Correction |
|-----------------|----------------|---|
| North | Rising | Add correction to observed bearing |
| North | Setting | Subtract correction from observed bearing |
| South | Rising | Subtract correction from observed bearing |
| South | Setting | Add correction to observed bearing |

Table 1503. Amplitude correction for the Sun.

The following two articles demonstrate the procedure for obtaining the amplitude of the Sun on both the celestial and visible horizons.

1504. Amplitude of the Sun on the Celestial Horizon

Mariners may use Bowditch [Table 22 \(Amplitudes\)](#) to determine the Sun's computed amplitude. The procedure is similar to that done in Section 1501. Comparing the computed amplitude to the amplitude measured with the gyrocompass determines the gyro error. In computing an amplitude, interpolate the tabular amplitude angle for the difference between the table arguments and the actual values of declination and latitude.

Do this double interpolation of the amplitude angle as

follows:

- Enter Bowditch [Table 22 \(Amplitudes\)](#) with the nearest integral values of declination and latitude. Extract a base amplitude angle.
- Reenter the table with the same declination argument but with the latitude to the next tabulated value (greater or less than the base latitude argument, depending upon whether the actual latitude is greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.
- Reenter the table with the base latitude argument but with the declination to the next tabulated value (greater or less than the base declination argument, depending upon whether the actual declination is greater or less than the base argument). Record the amplitude and the difference between it and the base amplitude angle and label it Diff.
- Compute the corrections due to latitude and declination not being exactly at a tabular value. Apply these corrections to obtain a final amplitude. The final amplitude is then converted to a true bearing. The difference between the true bearing and the gyro bearing gives the gyro error.

Example:

The DR latitude of a ship is 51° 24.6' N. The navigator observes the setting Sun on the celestial horizon. Its declination is N 19° 40.4'. Its observed bearing is 303° pgc.

Required:

Gyro error.

Solution:

Interpolate in [Table 22](#) for the Sun's calculated amplitude as follows. See [Figure 1504](#). The actual values for latitude and declination are L = 51.4° N and dec. = N 19.67°. Find the tabulated values of latitude and declination closest to these actual values. In this case, these tabulated values are L = 51° and dec. = 19.5°. Record the amplitude corresponding to these base values, 32.0°, as the base amplitude.

Next, holding the base declination value constant at 19.5°, increase the value of latitude to the next tabulated value: N 52°. Note that this value of latitude was increased because the actual latitude value was greater than the base value of latitude. Record the tabulated amplitude for L = 52° and dec. = 19.5°: 32.8°. Then, holding the base latitude value constant at 51°, increase the declination value to the next tabulated value: 20°. Record the tabulated amplitude for L = 51° and dec. = 20°: 32.9°.

The latitude's actual value (51.4°) is 0.4 of the way between the base value (51°) and the value used to determine the tabulated amplitude (52°). The declination's actual value (19.67°) is 0.3 of the way between the base value (19.5°) and the value used to determine the tabulated amplitude

(20.0°). To determine the total correction to base amplitude, multiply these increments (0.4 and 0.3) by the respective difference between the base and tabulated values (+0.8 and +0.9, respectively) and sum the products. The total correction is +0.6°. Add the total correction (+0.6°) to the base amplitude (32.0°) to determine the final amplitude (32.6°) which will be converted to a true bearing.

Because of its northerly declination (in this case), the Sun was 32.6° north of west when it was on the celestial horizon. Therefore its true bearing was 302.6° (270° + 32.6°) at this moment. Comparing this with the gyro bearing of 303° gives an error of 0.4°W, which can be rounded to 1/2°W.

| Actual | Base | Base Amp. | Tab. Amp. | Diff. | Inc. | Correction |
|-------------|-------|-----------|-----------|-------|------|--------------|
| L=51.4°N | 51° | 32.0° | 32.8° | +0.8° | 0.4 | +0.3° |
| dec=19.67°N | 19.5° | 32.0° | 32.9° | +0.9° | 0.3 | <u>+0.3°</u> |
| Total | | | | | | +0.6° |

Figure 1504. Interpolation in Table 22 for Amplitude.

1505. Amplitude of the Sun on the Visible Horizon

In higher latitudes, amplitude observations should be made when the body is on the visible horizon because the value of the correction is large enough to cause significant error if the observer misjudges the exact position of the celestial horizon. The observation will yield precise results whenever the visible horizon is clearly defined.

Example:

Observer's DR latitude is 59°47'N, Sun's declination is 5°11.3'S. At sunrise the Sun is observed on the visible horizon bearing 098.5° pgc.

Required:

Gyrocompass error.

Solution:

Given this particular latitude and declination, the amplitude angle is 10.3°S, so that the Sun's true bearing is 100.3° at the moment it is on the celestial horizon, that is, when its Hc is precisely 0°. Applying the Table 23 correction to the observed bearing of 098.5° pgc using the rules given in Section 1503, the Sun would have been bearing 099.7° pgc had the observation been made when the Sun was on the celestial horizon. Therefore, the gyro error is 0.6°E.

1506. Amplitude by Calculation

As an alternative to using the amplitude tables, if a calculator is available then the amplitudes can be computed using a slightly modified version of the altitude-azimuth formula. The modification is needed because azimuth (Z) and amplitude (A) angles are complimentary, and the co-functions of complimentary angles are equal; i.e., cosine Z = sine A. In the following formulas, northerly latitudes and declinations are given positive values, and southerly latitudes and declinations are considered negative. If the

resulting amplitude is positive, it is north of the prime vertical; conversely, a negative amplitude is south of the prime vertical.

- a) The general case, when a body is not on the celestial horizon, the formula is:

$$\text{Amplitude} = \sin^{-1}[(\sin \text{DEC} - \sin \text{LAT} \sin \text{Hc}) / (\cos \text{LAT} \cos \text{Hc})]$$

where DEC is the celestial body's declination, LAT is the observer's latitude, and Hc is the object's computed altitude. For the Sun on the visible horizon, Hc = -0.7°.

- b) When a body is on the celestial horizon (that is, its altitude, Hc = 0), the formula becomes:

$$\text{Amplitude} = \sin^{-1} [\sin \text{DEC} / \cos \text{LAT}]$$

Example:

Determine the gyrocompass error using the formulation instead of the tables, for the example in Section 1505.

Required:

Gyrocompass error.

Solution:

The observed bearing of the Sun on the visible horizon is 098.5° pgc. The computed amplitude of the Sun when it is on the visible horizon (that is, Hc = -0.7°) is found by:

$$\text{Amplitude} = \sin^{-1} [(\sin -5.19^\circ - \sin 59.78^\circ \sin -0.7^\circ) / (\cos 59.78^\circ \cos -0.7^\circ)].$$

Evaluating, we find the amplitude is 9.1°. This is 9.1° degrees away from E, in the "negative" (or southerly) direction, so the calculated azimuth is 90° + 9.1° = 99.1°. The gyrocompass error is 99.1° - 98.5° = 0.6° E. This value matches the answer obtained in Section 1505 using the tables.